High School Mathematics
Geometry Vocabulary Word Wall Cards

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Triangles

Polygons and Circles

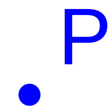
Three-Dimensional Figures

Revision:
November 2014 – Contrapositive card - corrected “converse” to “contrapositive” in example

Reasoning, Lines, and Transformations

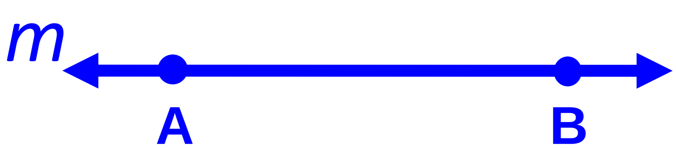
Basics of Geometry

Point – A point has no dimension. It is a location on a plane. It is represented by a dot.



point P

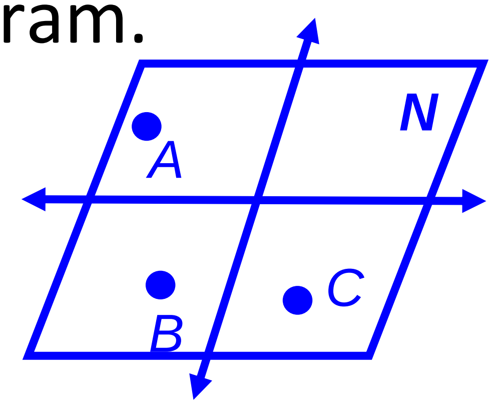
Line – A line has one dimension. It is an infinite set of points represented by a line with two arrowheads that extends without end.



$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ or line *m*

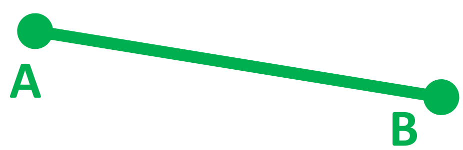
Plane – A plane has two dimensions extending without end. It is often represented by a parallelogram.

plane ABC or plane *N*



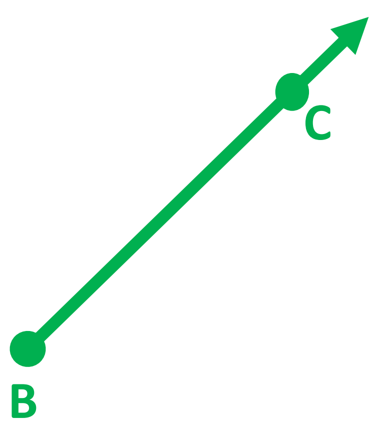
Basics of Geometry

Line segment – A line segment consists of two endpoints and all the points between them.



$\overline{AB}$ or $\overline{BA}$

Ray – A ray has one endpoint and extends without end in one direction.



$\overrightarrow{BC}$

Note: Name the endpoint first.
$\overrightarrow{BC}$ and $\overrightarrow{CB}$ are different rays.

Geometry Notation

Symbols used to represent statements or operations in geometry.

$\overline{BC}$	segment BC
$\overrightarrow{BC}$	ray BC
$\overleftrightarrow{BC}$	line BC
BC	length of BC
$\angle ABC$	angle ABC
$m\angle ABC$	measure of angle ABC
ΔABC	triangle ABC
$\|\|$	is parallel to
$\perp$	is perpendicular to
$\cong$	is congruent to
$\sim$	is similar to

Logic Notation

$\vee$	or
$\wedge$	and
$\rightarrow$	read "implies", if… then…
$\leftrightarrow$	read "if and only if"
iff	read "if and only if"
~	not
$\therefore$	therefore

Set Notation

{}	empty set, null set
Ø	empty set, null set
***x* |**	read “*x* such that”
***x* :**	read “*x* such that”
∪	union, disjunction, or
∩	intersection, conjunction, and

Conditional Statement

a logical argument consisting of
a set of premises,
hypothesis (p), and conclusion (q)

hypothesis

If an angle is a right angle,
then its measure is 90°.

conclusion

Symbolically:

if p, then q $\qquad$ $p \rightarrow q$

Converse

formed by interchanging the hypothesis and conclusion of a conditional statement

Conditional: If an angle is a right angle, then its measure is 90°.

Converse: If an angle measures 90°, then the angle is a right angle.

Symbolically:

if q, then p $\quad\quad$ $q \rightarrow p$

Inverse

formed by negating the hypothesis and conclusion of a conditional statement

Conditional: If an angle is a right angle, then its measure is 90°.

Inverse: If an angle is not a right angle, then its measure is not 90°.

Symbolically:

if ~p, then ~q $\qquad$ ~p→~q

Contrapositive

formed by interchanging and negating the hypothesis and conclusion of a conditional statement

Conditional: If an angle is a right angle, then its measure is 90°.

Contrapositive: If an angle does not measure 90°, then the angle is not a right angle.

Symbolically:

if ~q, then ~p   ~q → ~p

Symbolic Representations

Conditional	if p, then q	$p \rightarrow q$
Converse	if q, then p	$q \rightarrow p$
Inverse	if not p, then not q	$\sim p \rightarrow \sim q$
Contrapositive	if not q, then not p	$\sim q \rightarrow \sim p$

Deductive Reasoning

method using logic to draw conclusions based upon definitions, postulates, and theorems

Example:

Prove $(x \cdot y) \cdot z = (z \cdot y) \cdot x$.

Step 1: $(x \cdot \mathrm{y}) \cdot z = z \cdot (x \cdot \mathrm{y})$, using commutative property of multiplication.

Step 2: $= z \cdot (y \cdot x)$, using commutative property of multiplication.

Step 3: $= (z \cdot y) \cdot x$, using associative property of multiplication.

Inductive Reasoning

method of drawing conclusions from a limited set of observations

Example:
Given a pattern, determine the rule for the pattern.

Determine the next number in this sequence 1, 1, 2, 3, 5, 8, 13…

Proof

a justification logically valid and based on initial assumptions, definitions, postulates, and theorems

Example:

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 2 \cong \angle 1$

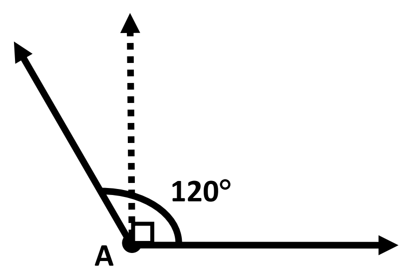
Statements	Reasons
$\angle 1 \cong \angle 2$	Given
$m\angle 1 = m\angle 2$	Definition of congruent angles
$m\angle 2 = m\angle 1$	Symmetric Property of Equality
$\angle 2 \cong \angle 1$	Definition of congruent angles

Properties of Congruence

Reflexive Property	For all angles *A*, $\angle A \cong \angle A$. An angle is congruent to itself.
Symmetric Property	For any angles *A* and *B*, If $\angle A \cong \angle B$, then $\angle B \cong \angle A$. Order of congruence does not matter.
Transitive Property	For any angles *A*, *B*, and *C*, If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. If two angles are both congruent to a third angle, then the first two angles are also congruent.

Law of Detachment

deductive reasoning stating that if the hypothesis of a true conditional statement is true then the conclusion is also true



Example:

If $m\angle A > 90°$, then $\angle A$ is an obtuse angle. $m\angle A = 120°$.

Therefore, $\angle A$ is an obtuse angle.

If $p \rightarrow q$ is a true conditional statement and p is true, then q is true.

Law of Syllogism

deductive reasoning that draws a new conclusion from two conditional statements when the conclusion of one is the hypothesis of the other

Example:

1. If a rectangle has four equal side lengths, then it is a square.
2. If a polygon is a square, then it is a regular polygon.
3. If a rectangle has four equal side lengths, then it is a regular polygon.

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then $p \rightarrow r$ is true.

Counterexample

specific case for which a conjecture is false

Example:

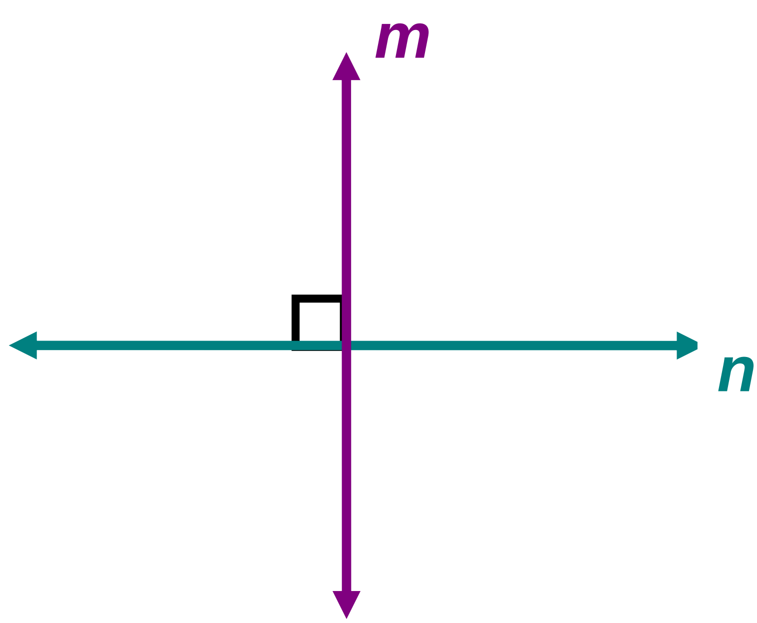
Conjecture: “The product of any two numbers is odd.”

Counterexample: **$2 \cdot 3 = 6$**

<u>One</u> counterexample proves a conjecture false.

Perpendicular Lines

two lines that intersect to form a right angle

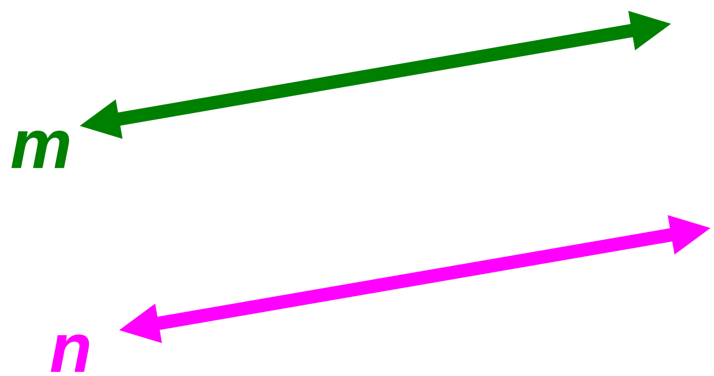


Line *m* is perpendicular to line *n*.

$m \perp n$

Parallel Lines

lines that do not intersect and are coplanar



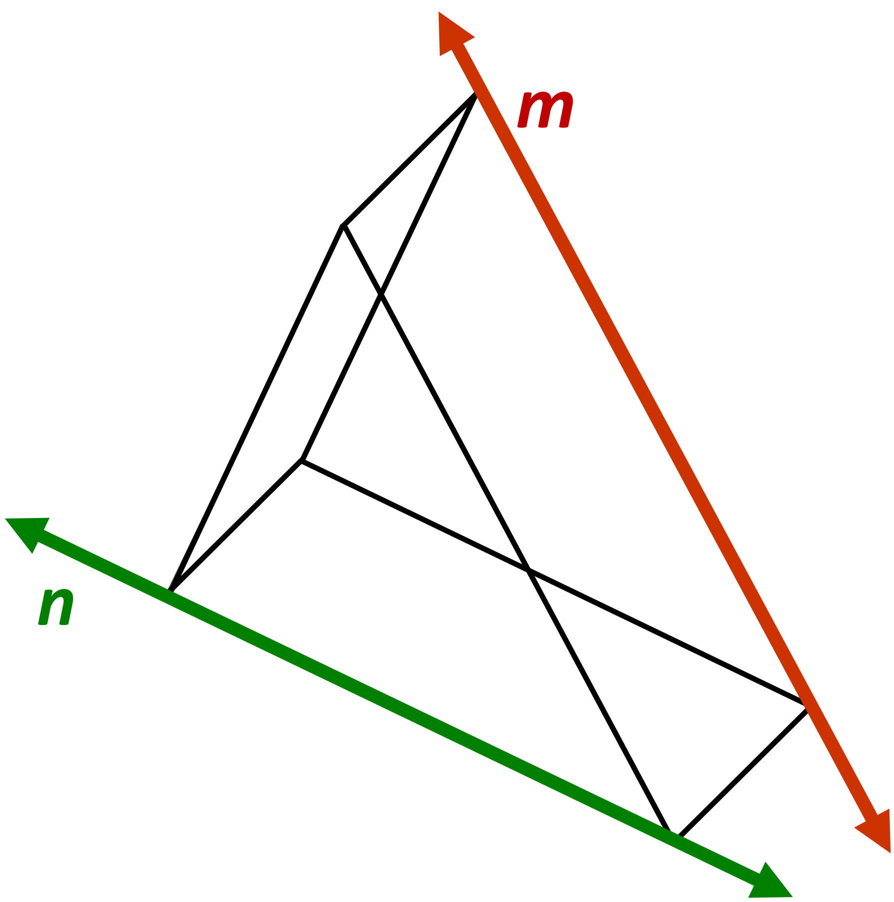
$m || n$

Line *m* is parallel to line *n.*

Parallel lines have the same slope.

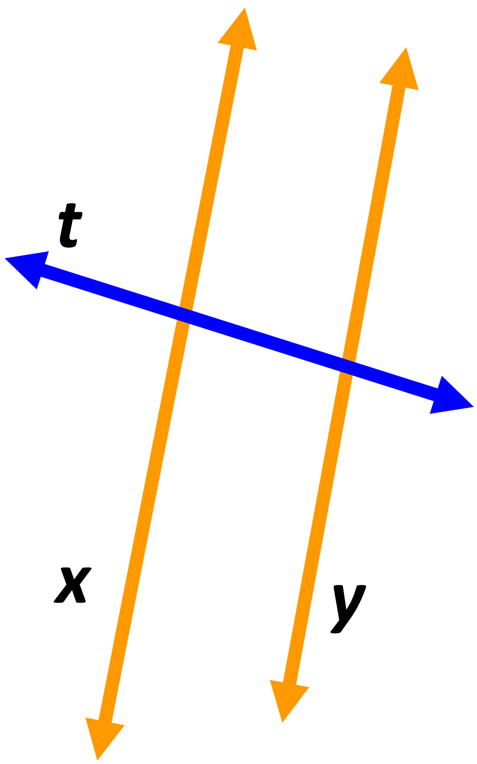
Skew Lines

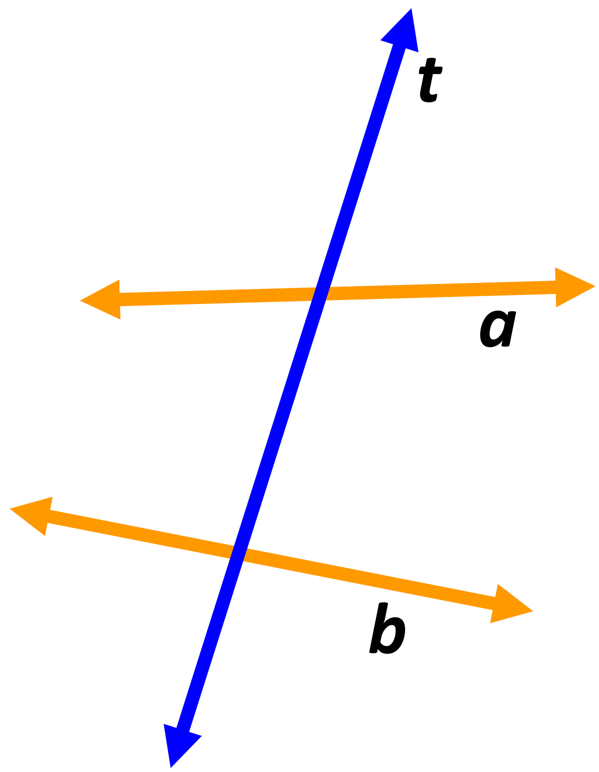
lines that do not intersect and are not coplanar



Transversal

a line that intersects at least two other lines

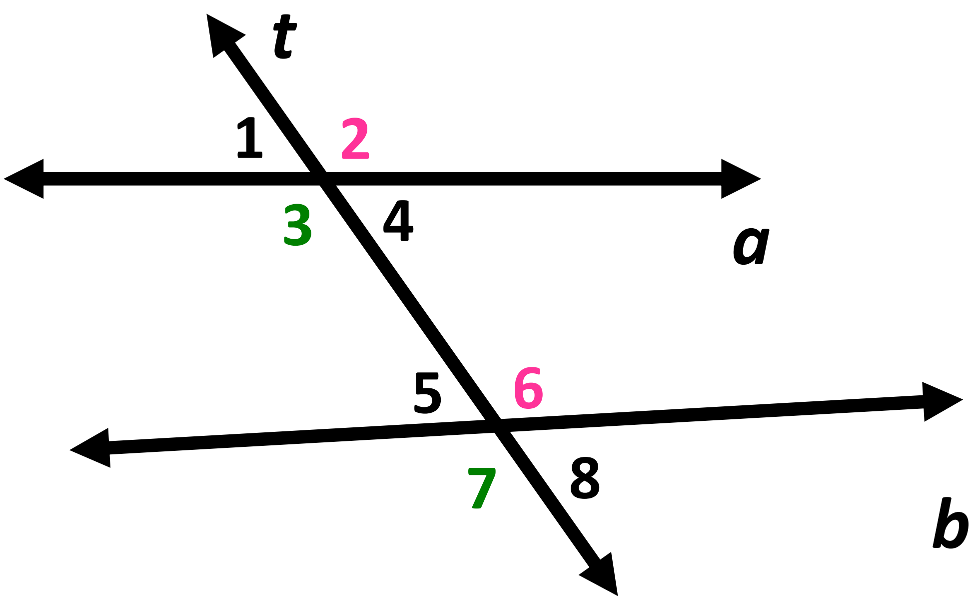




Line *t* is a transversal.

Corresponding Angles

angles in matching positions when a transversal crosses at least two lines

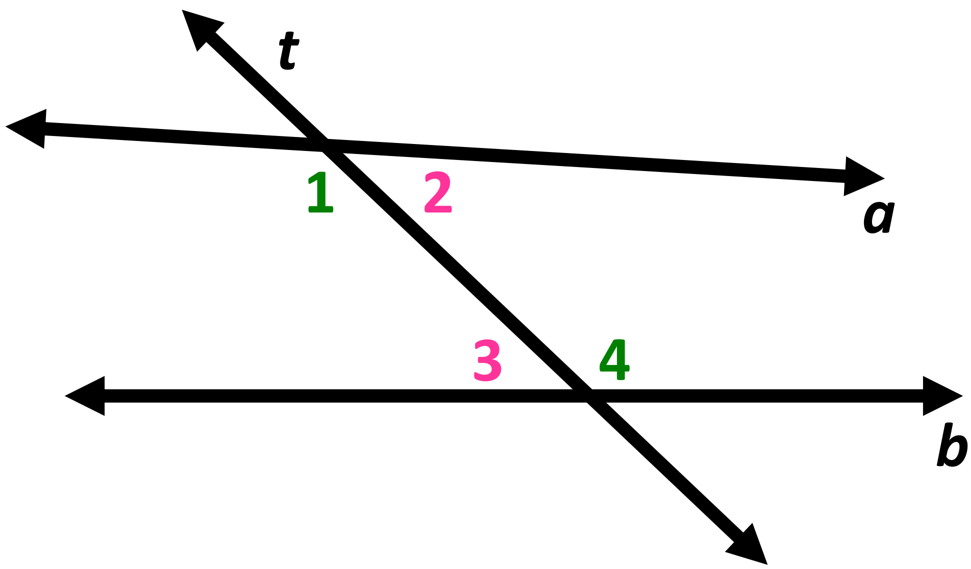


Examples:

1) $\angle 2$ and $\angle 6$
2) $\angle 3$ and $\angle 7$

Alternate Interior Angles

angles inside the lines and on opposite sides of the transversal

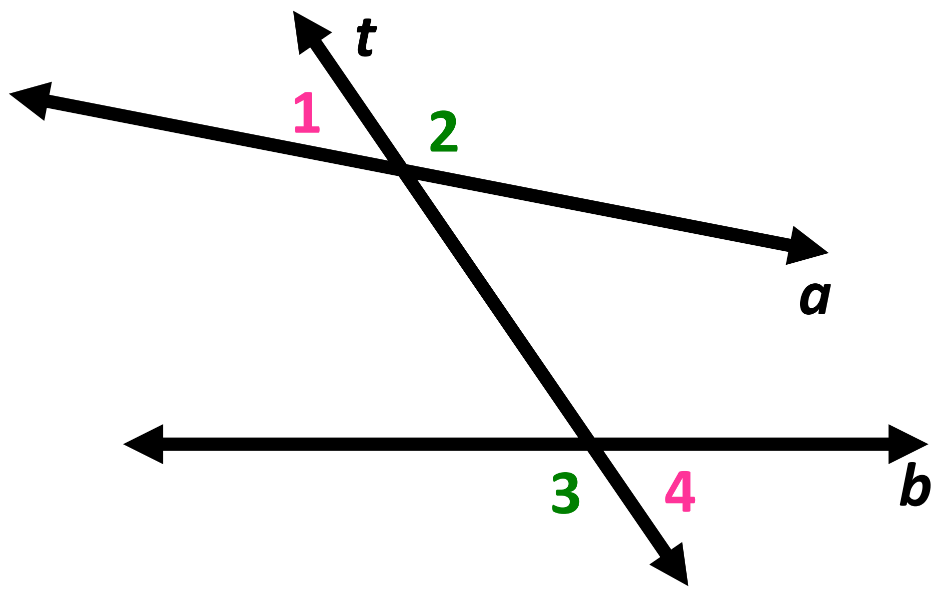


Examples:

1) $\angle 1$ and $\angle 4$
2) $\angle 2$ and $\angle 3$

Alternate Exterior Angles

angles outside the two lines and on opposite sides of the transversal

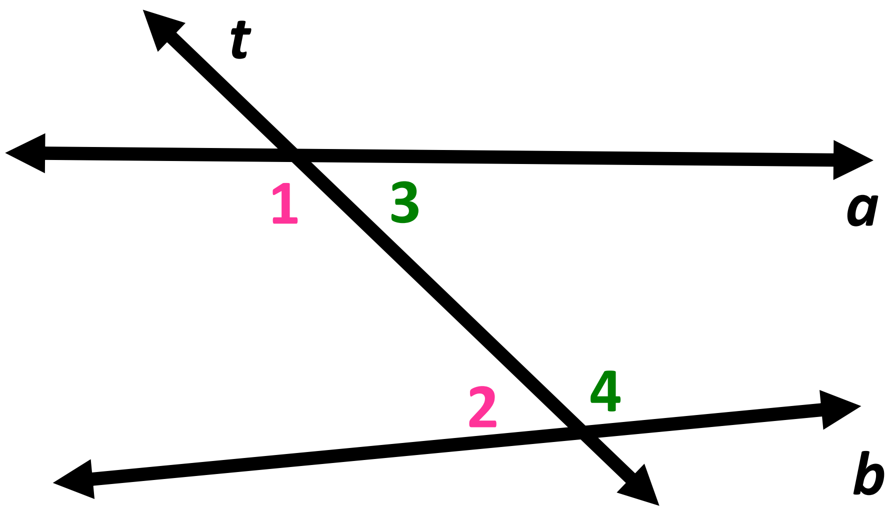


Examples:

1) $\angle 1$ and $\angle 4$
2) $\angle 2$ and $\angle 3$

Consecutive Interior Angles

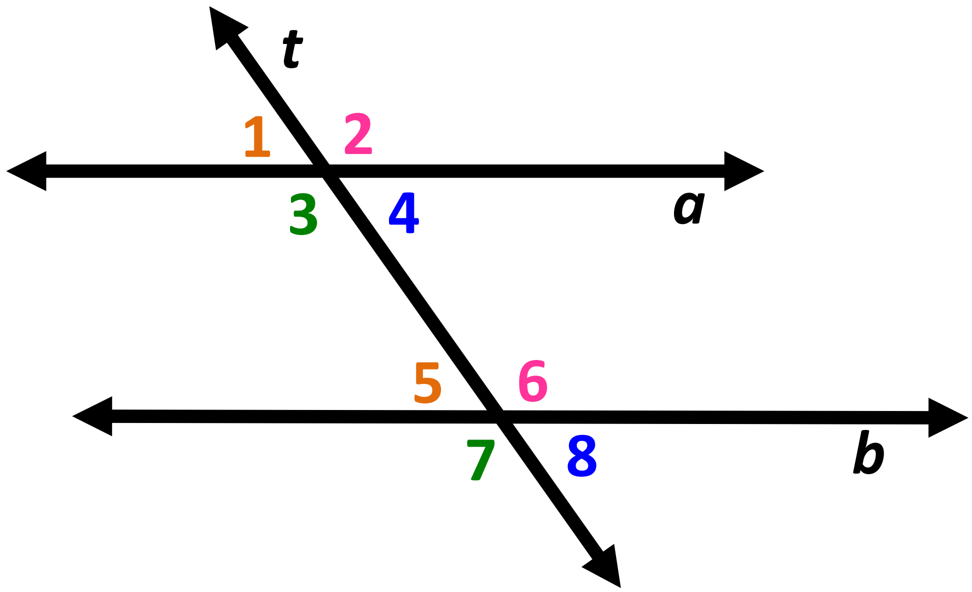
angles between the two lines and on the same side of the transversal



Examples:

1) $\angle 1$ and $\angle 2$
2) $\angle 3$ and $\angle 4$

Parallel Lines

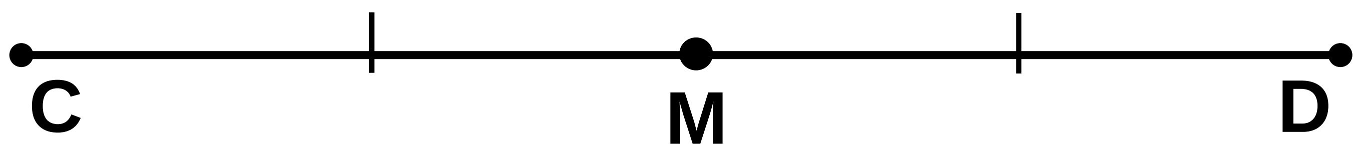


Line *a* is parallel to line *b* when

Corresponding angles are congruent	$\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$
Alternate interior angles are congruent	$\angle 3 \cong \angle 6$ $\angle 4 \cong \angle 5$
Alternate exterior angles are congruent	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$
Consecutive interior angles are supplementary	$m\angle 3 + m\angle 5 = 180°$ $m\angle 4 + m\angle 6 = 180°$

Midpoint

divides a segment into two congruent segments



Example: M is the midpoint of $\overline{CD}$

$\overline{CM} \cong \overline{MD}$

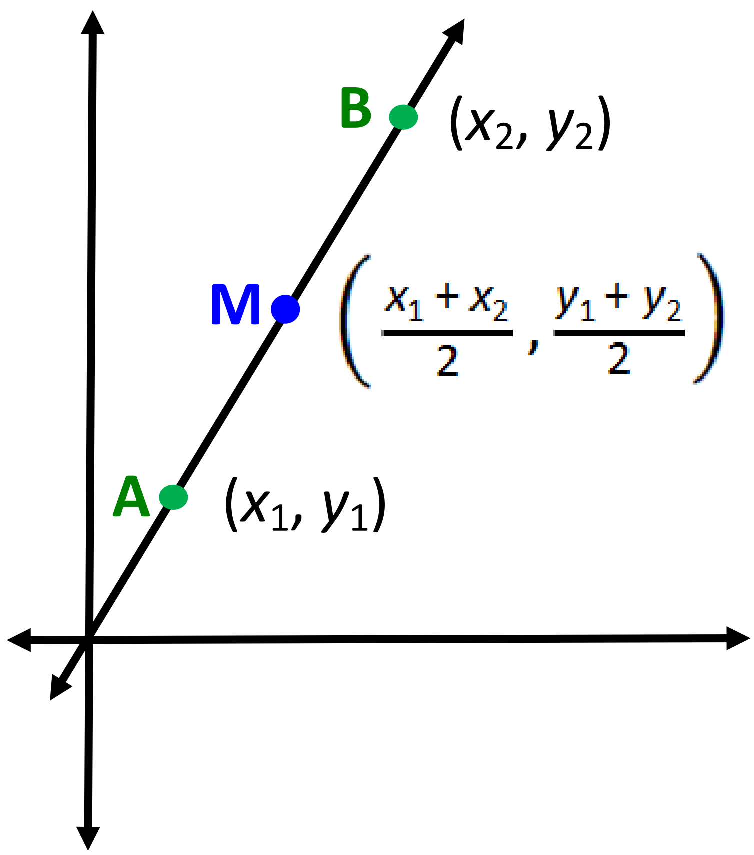
$CM = MD$

Segment bisector may be a point, ray, line, line segment, or plane that intersects the segment at its midpoint.

Midpoint Formula

given points A(x_1, y_1) and B(x_2, y_2)

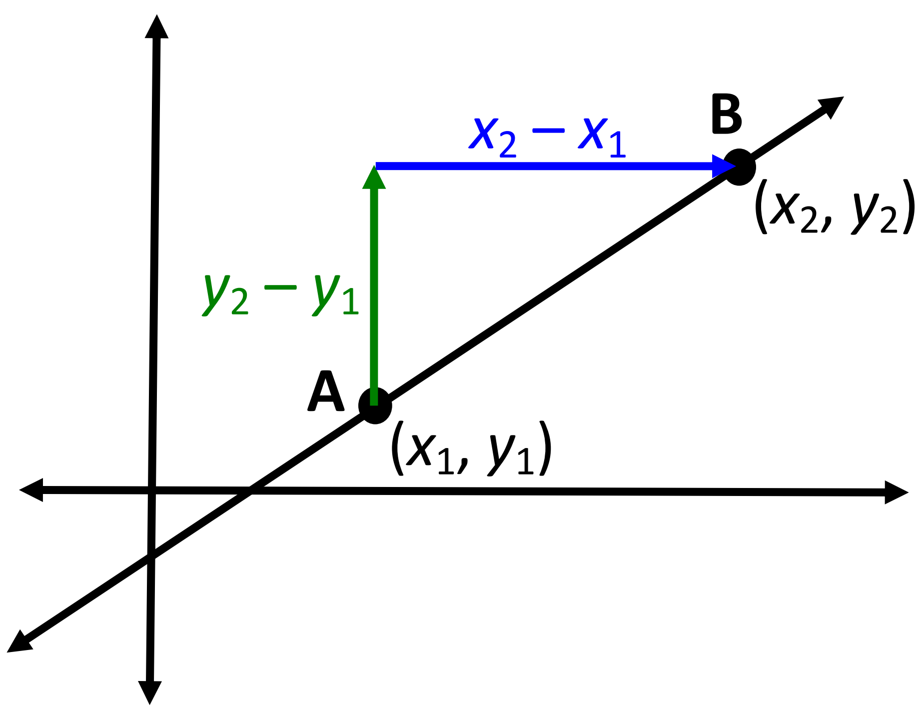
midpoint M = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



Slope Formula

ratio of vertical change to horizontal change

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



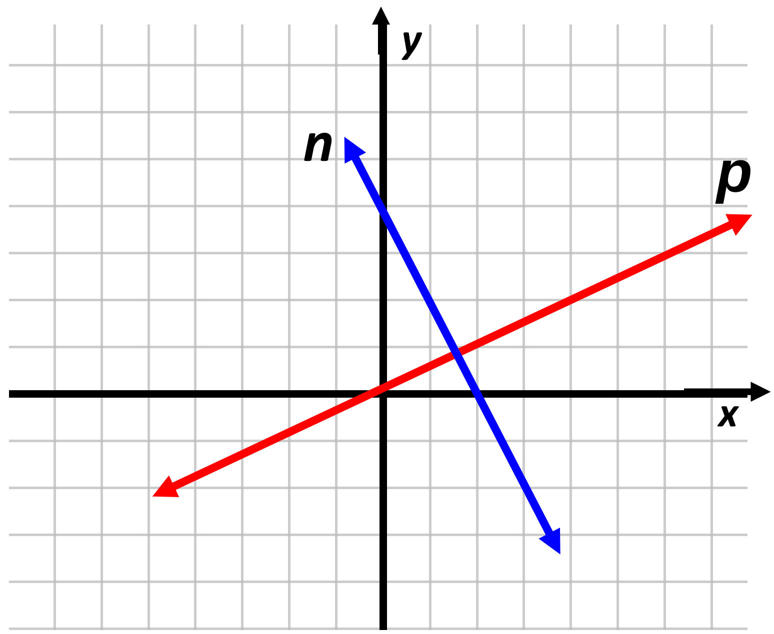
Slopes of Lines

Parallel lines have the same slope.

Perpendicular lines have slopes whose product is -1.

Vertical lines have undefined slope.

Horizontal lines have 0 slope.



Example:

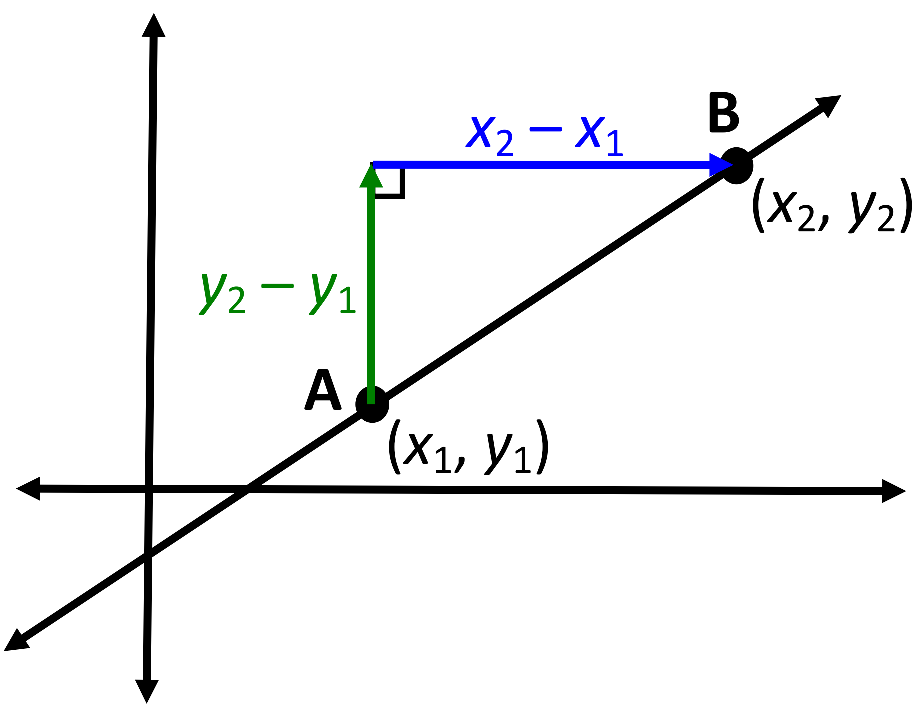
The slope of line n = -2. The slope of line $p = \frac{1}{2}$.

$$-2 \cdot \frac{1}{2} = -1\text{, therefore, } n \perp p.$$

Distance Formula

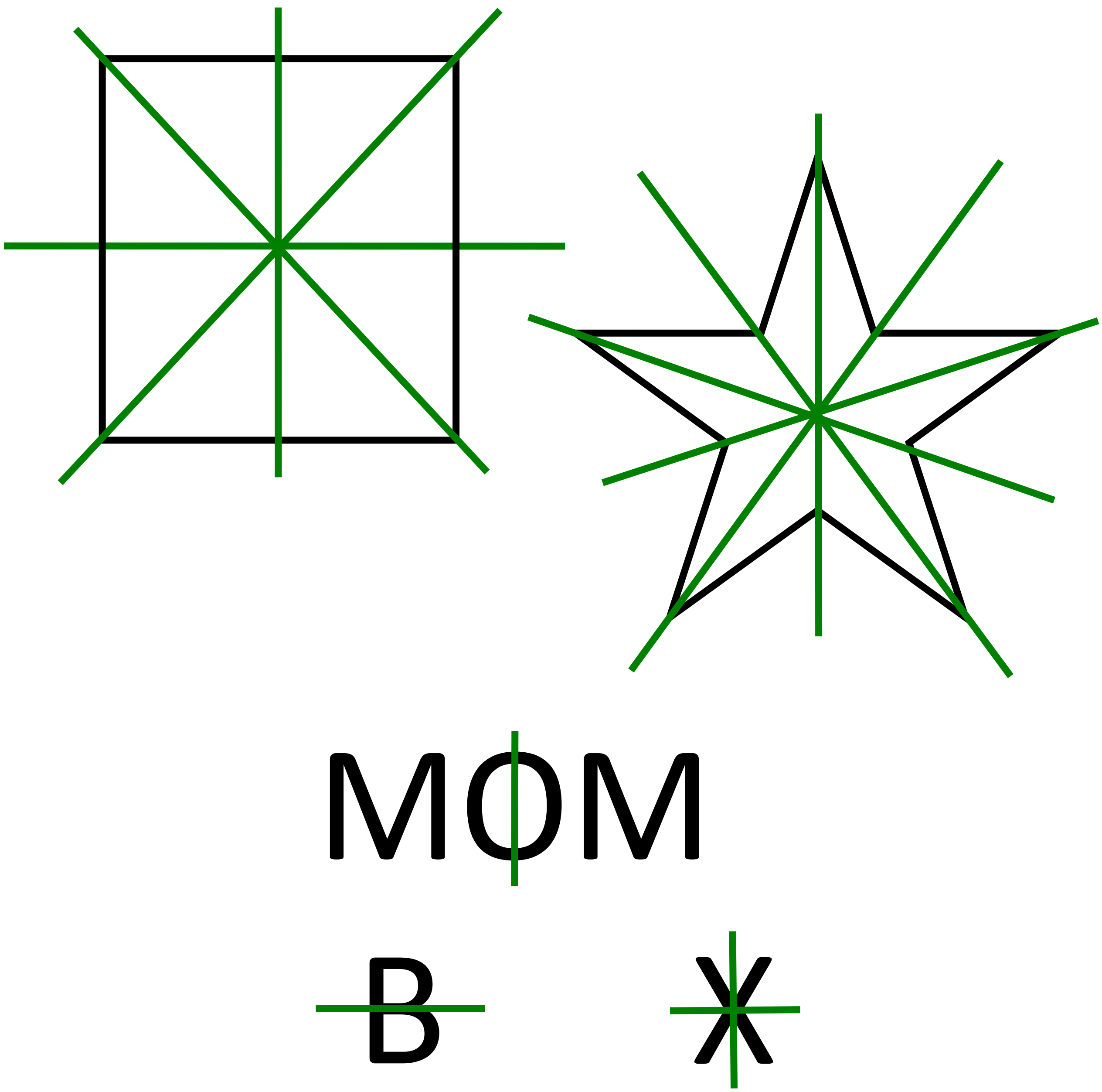
given points A (x_1, y_1) and B (x_2, y_2)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

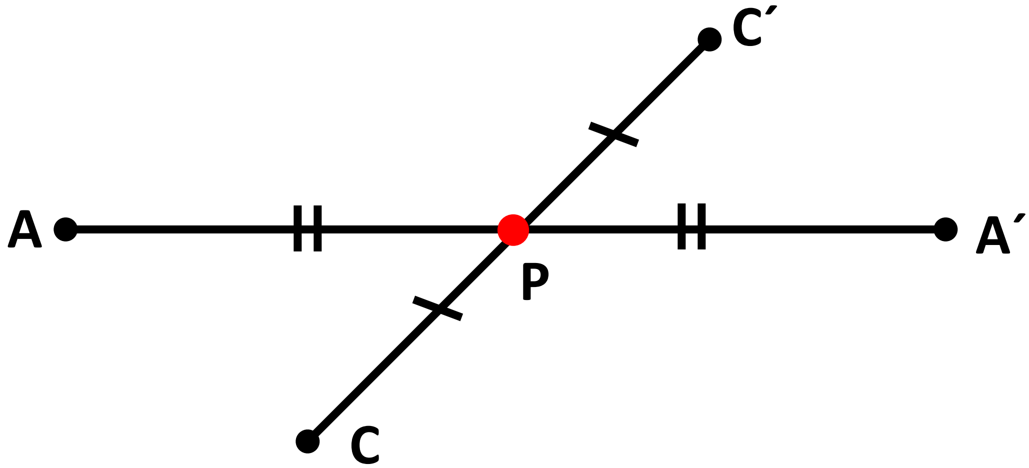


The distance formula is based on the Pythagorean Theorem.

Line Symmetry

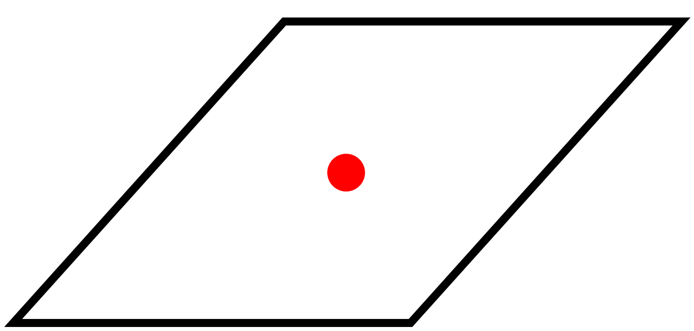


Point Symmetry

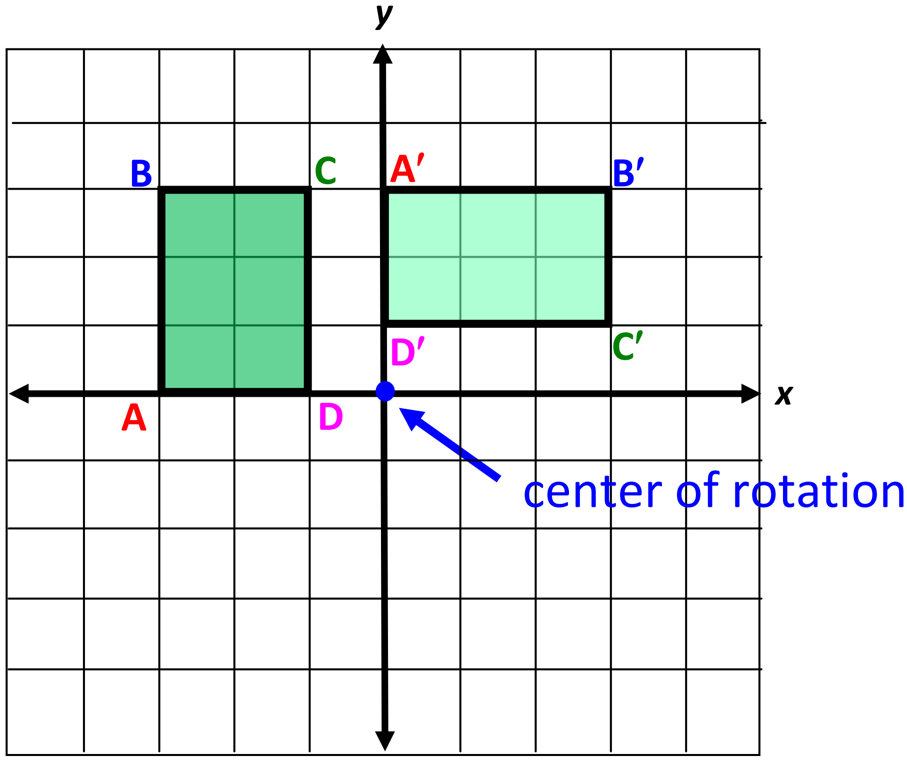


pod

S Z



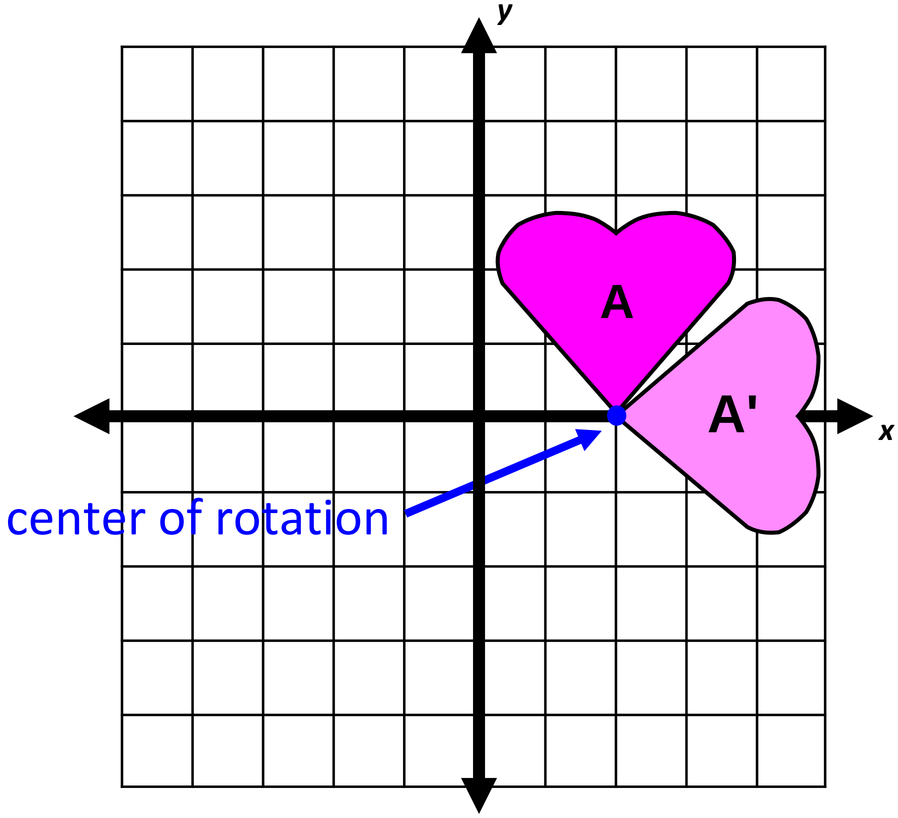
Rotation



Preimage	Image
A(-3,0)	A′(0,3)
B(-3,3)	B′(3,3)
C(-1,3)	C′(3,1)
D(-1,0)	D′(0,1)

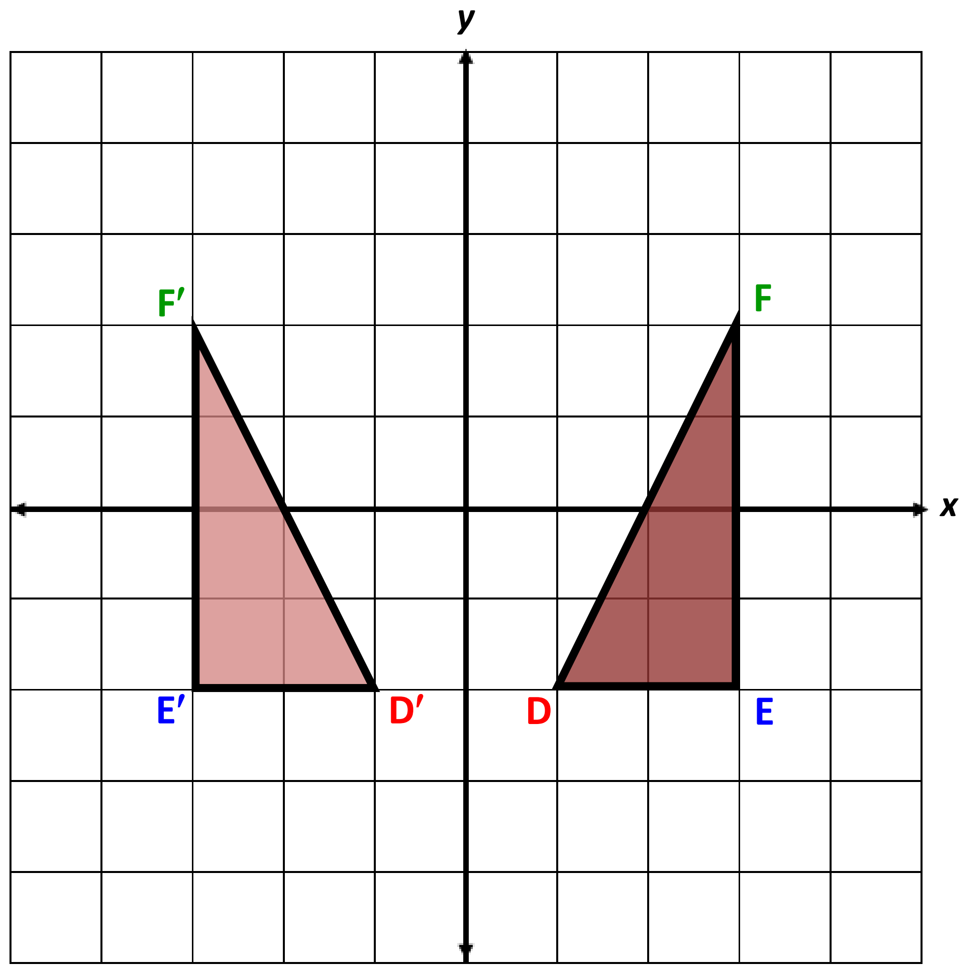
Pre-image has been transformed by a 90° clockwise rotation about the origin.

Rotation



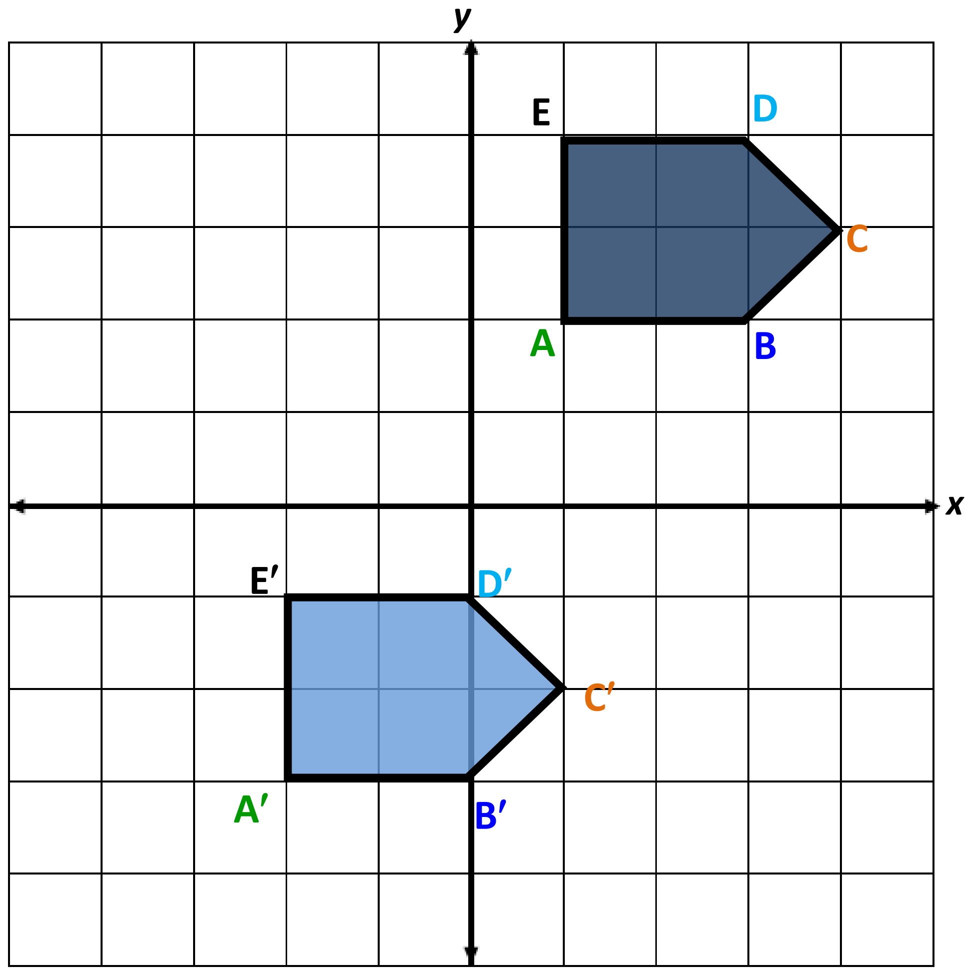
Pre-image A has been transformed by a 90° clockwise rotation about the point (2, 0) to form image A$^{\prime}$.

Reflection



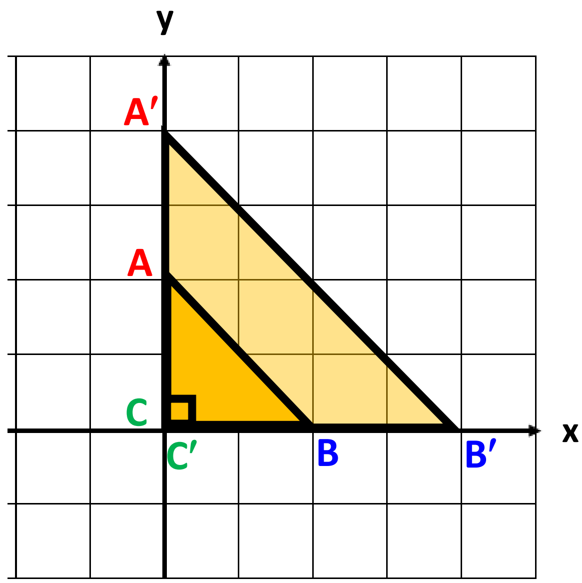
Preimage	Image
D(1,-2)	D′(-1,-2)
E(3,-2)	E′(-3,-2)
F(3,2)	F′(-3,2)

Translation



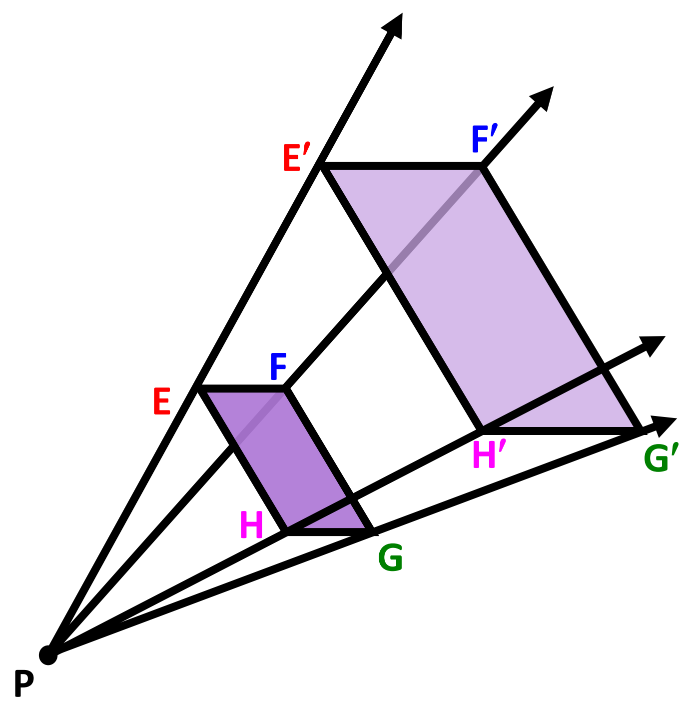
Preimage	Image
A(1,2)	A′(-2,-3)
B(3,2)	B′(0,-3)
C(4,3)	C′(1,-2)
D(3,4)	D′(0,-1)
E(1,4)	E′(-2,-1)

Dilation



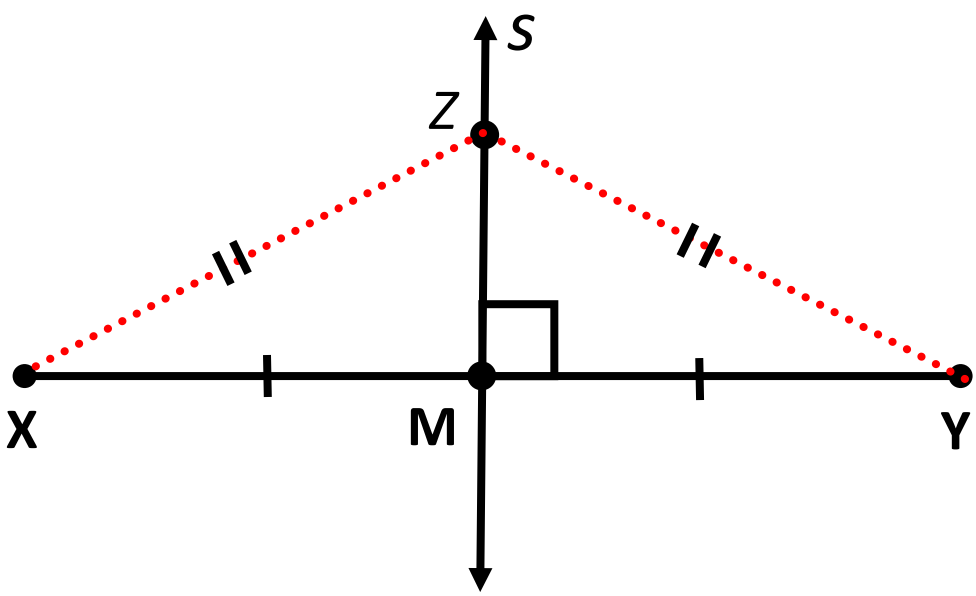
Preimage	Image
A(0,2)	A′(0,4)
B(2,0)	B′(4,0)
C(0,0)	C′(0,0)

Preimage	Image
E	E′
F	F′
G	G′
H	H′



Perpendicular Bisector

a segment, ray, line, or plane that is perpendicular to a segment at its midpoint



Example:

Line *s* is perpendicular to $\overline{XY}$.

M is the midpoint, therefore $\overline{XM} \cong \overline{MY}$.

Z lies on line *s* and is equidistant from X and Y.

Constructions

Traditional constructions involving a compass and straightedge reinforce students' understanding of geometric concepts. Constructions help students visualize Geometry.

There are multiple methods to most geometric constructions. These cards illustrate only one method. Students would benefit from experiences with more than one method and should be able to justify each step of geometric constructions.

Construct

segment *CD* congruent to segment *AB*

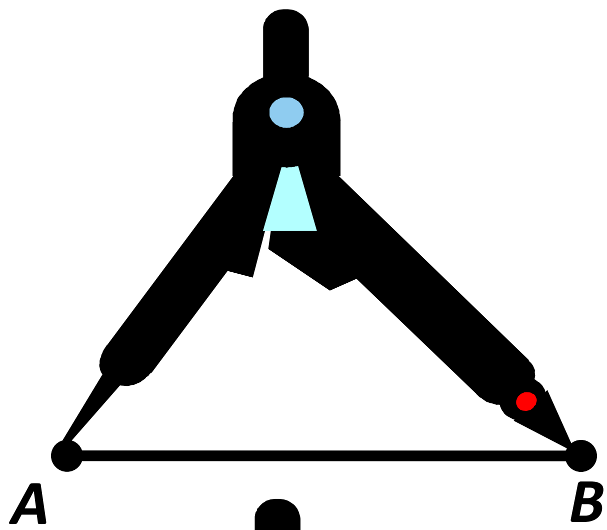


Fig. 1

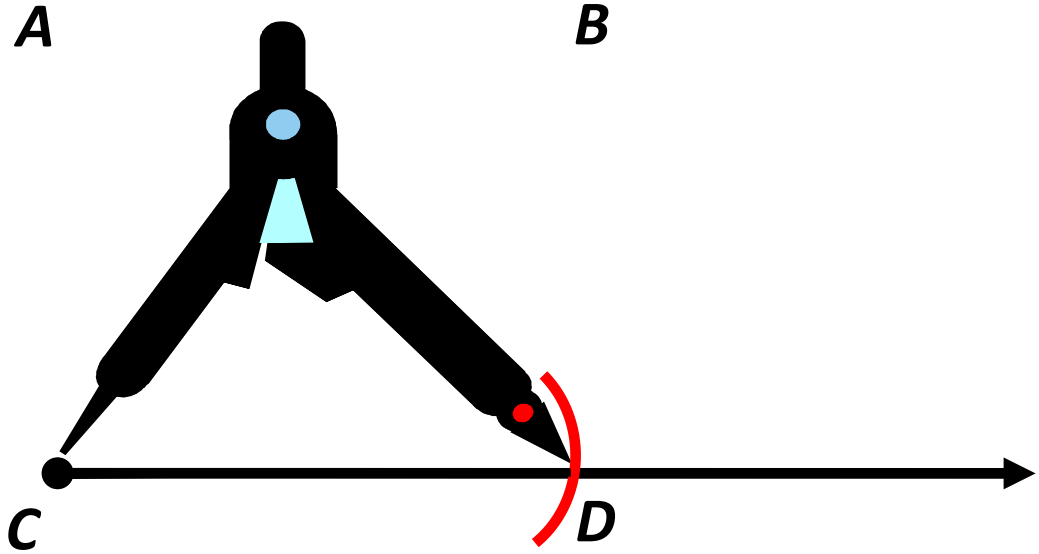


Fig. 2

Construct

a perpendicular bisector of segment *AB*

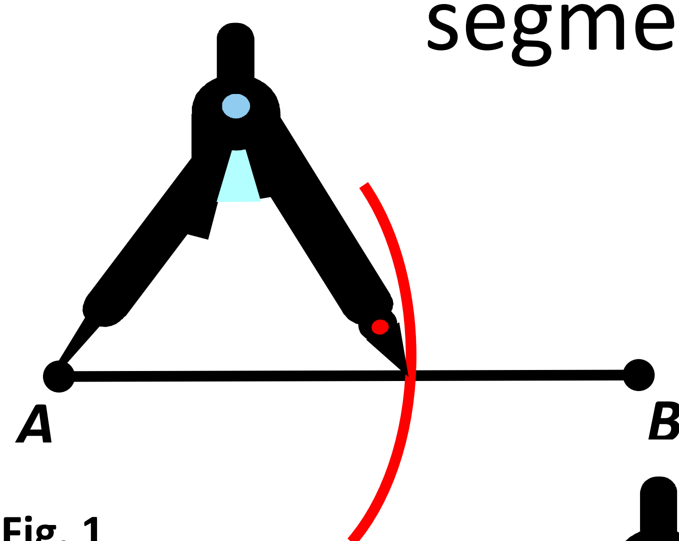


Fig. 1

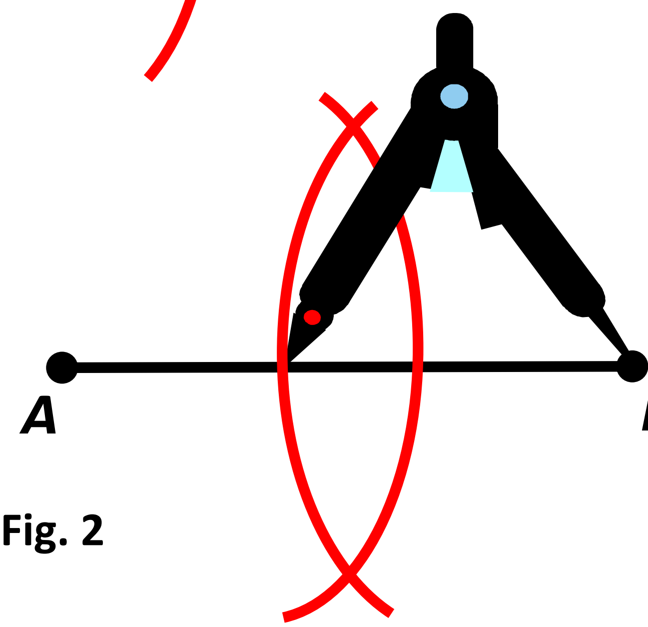


Fig. 2

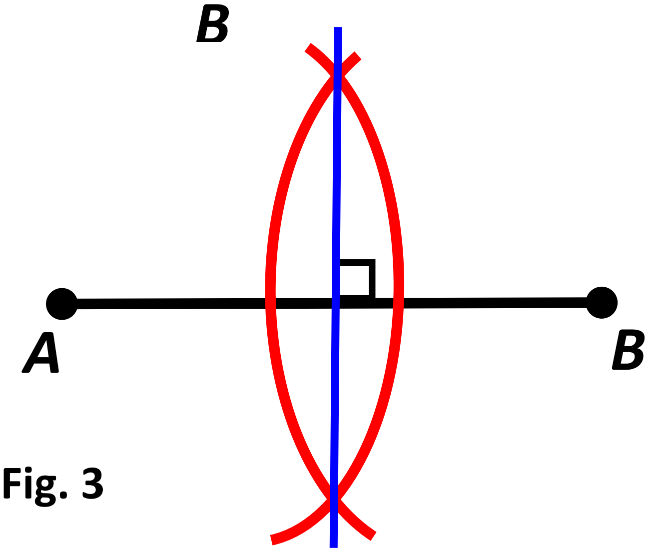


Fig. 3

Construct

a perpendicular to a line from point P not on the line

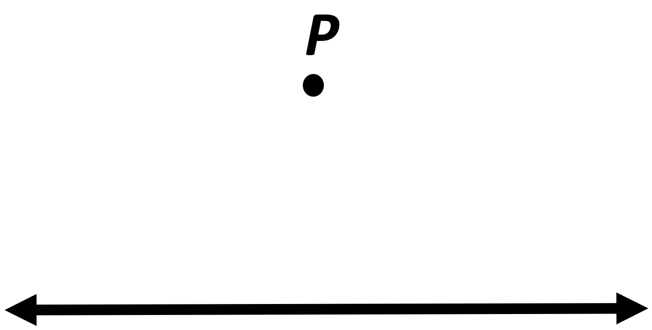


Fig. 1

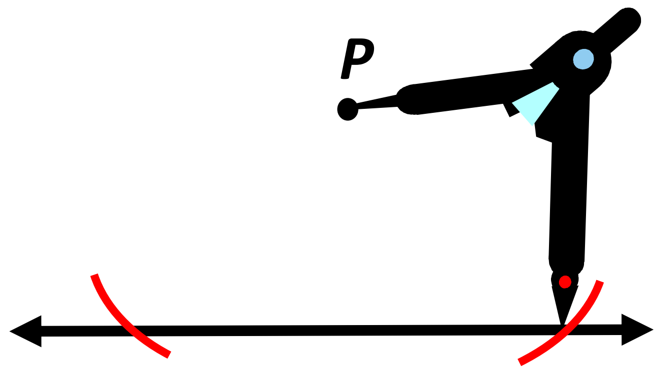


Fig. 2

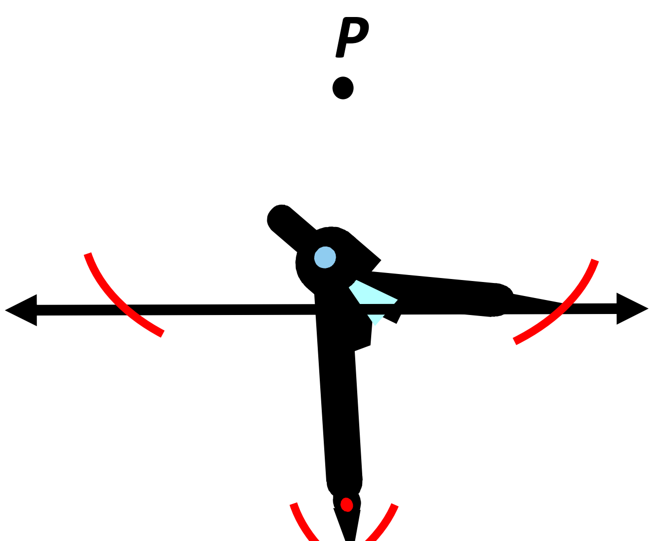


Fig. 3

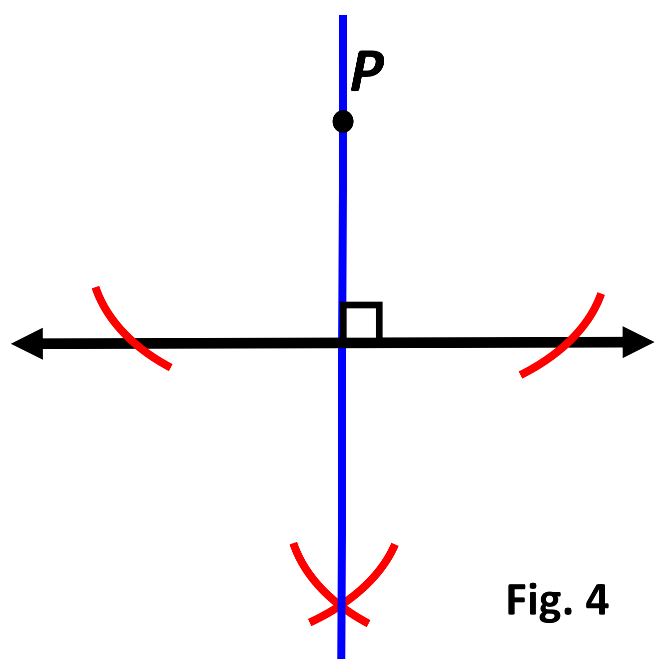


Fig. 4

Construct

a perpendicular to a line from point P on the line

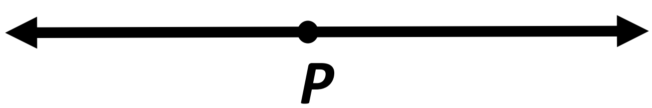


Fig. 1

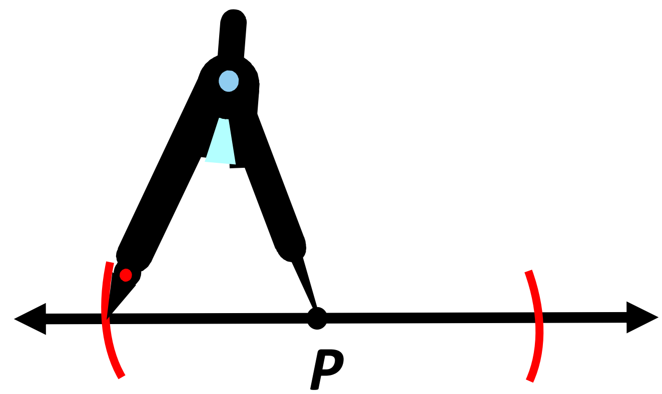


Fig. 2

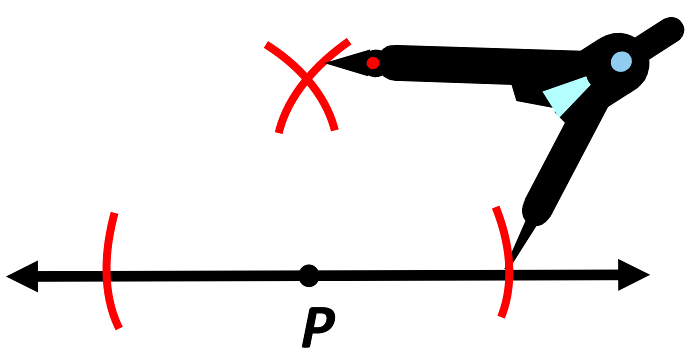


Fig. 3

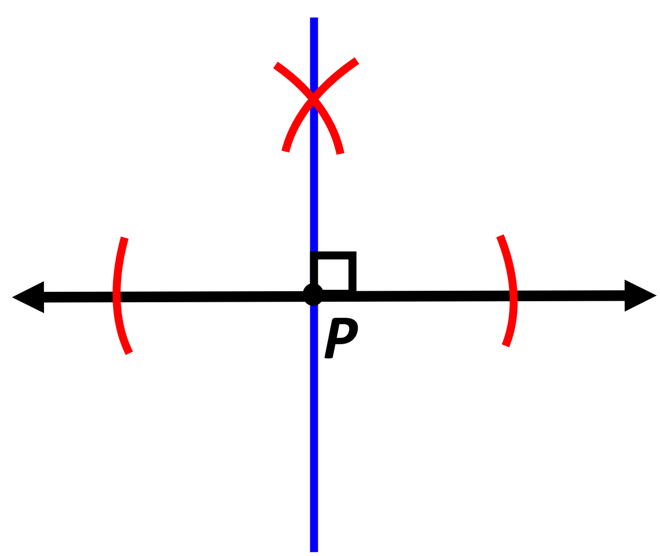


Fig. 4

Construct

a bisector of $\angle A$

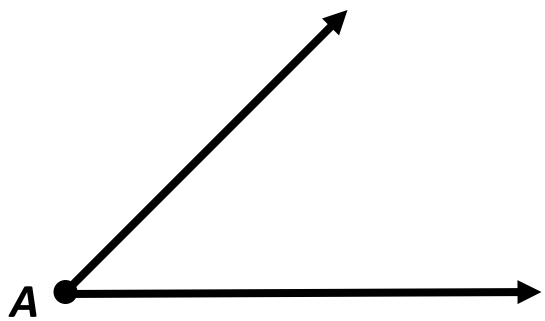


Fig. 1

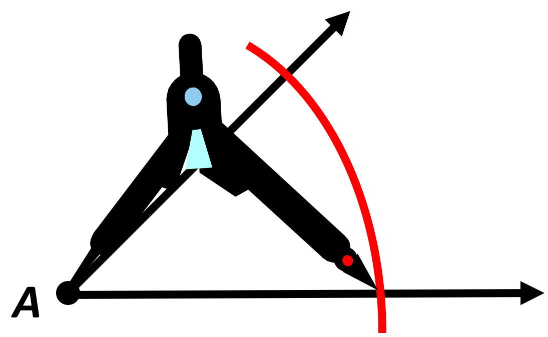


Fig. 2

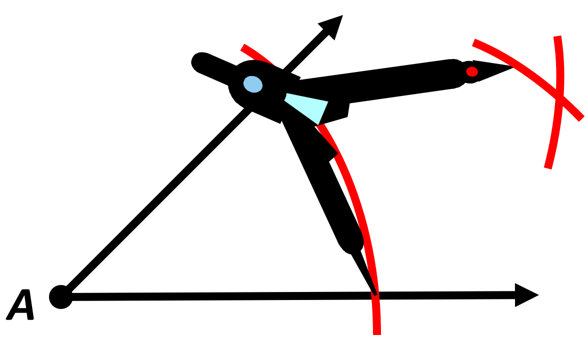


Fig. 3

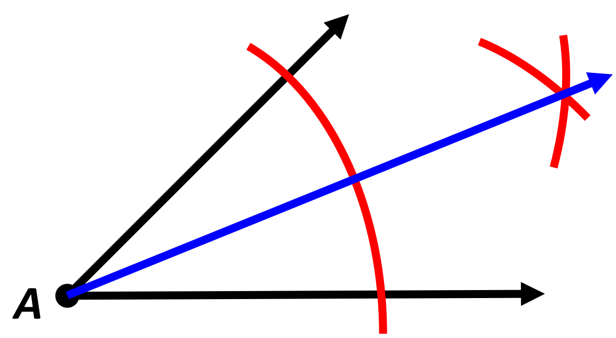


Fig. 4

Construct

$\angle Y$ congruent to $\angle A$

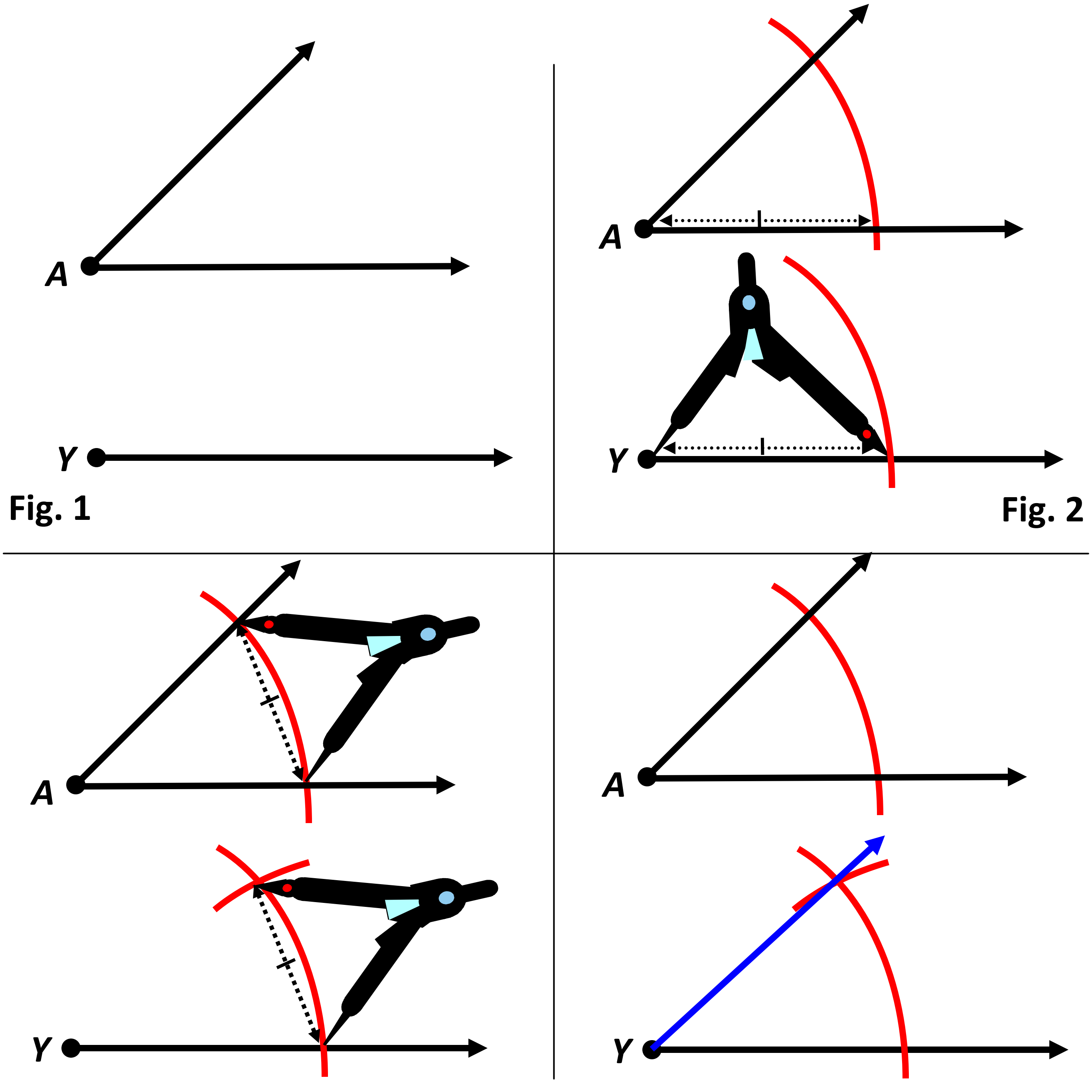
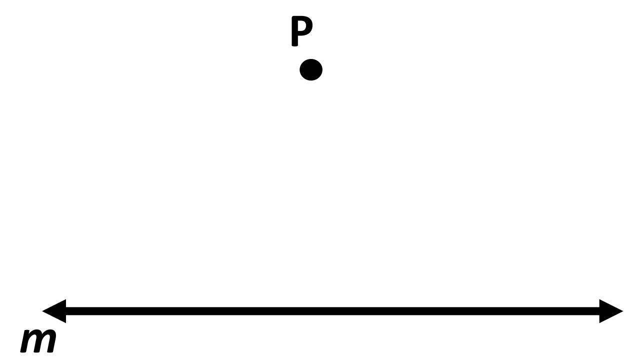


Fig. 1 Fig. 2 Fig. 3 Fig. 4

Construct

line *n* parallel to line *m* through point *P* not on the line



Draw a line through point P intersecting line m.

Fig. 1

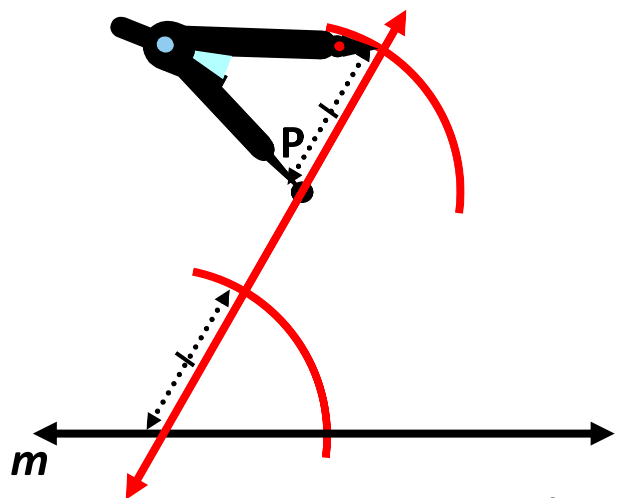


Fig. 2

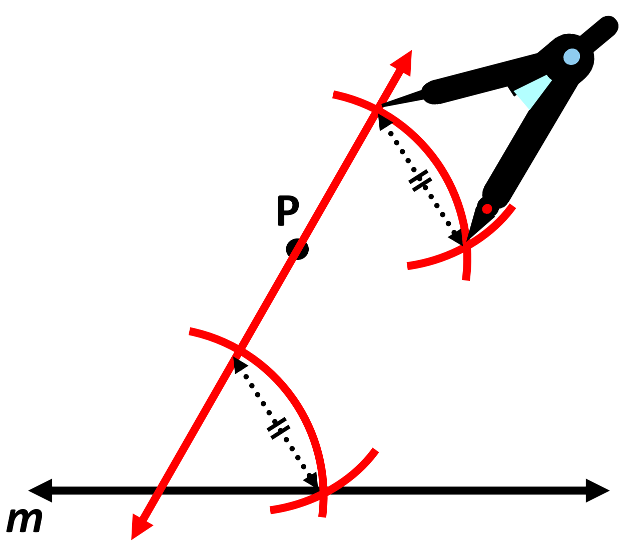


Fig. 3

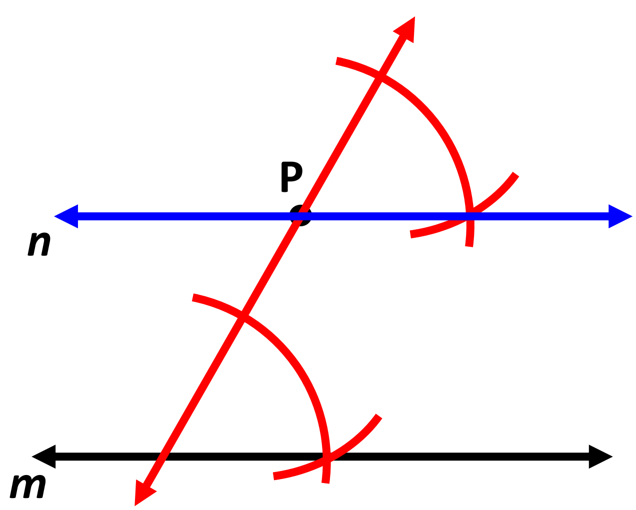


Fig. 4

Construct

an equilateral triangle inscribed in a circle

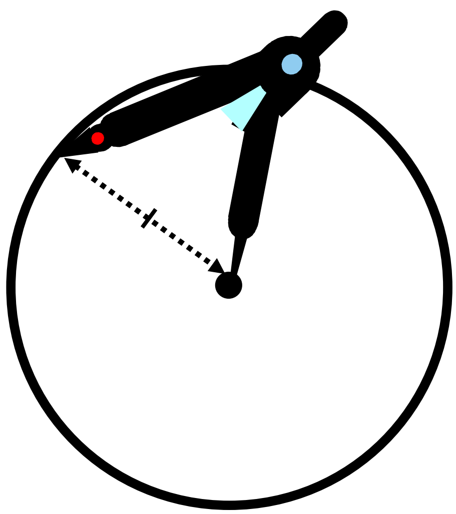


Fig. 1

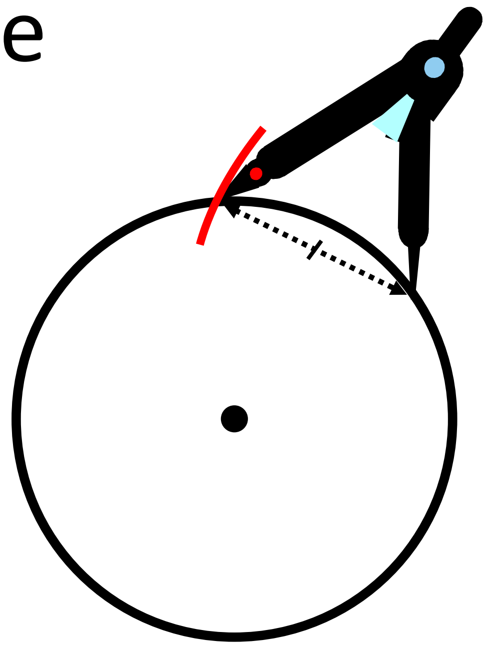


Fig. 2

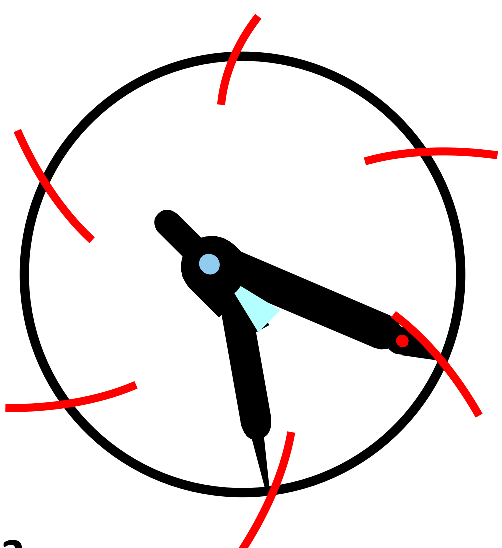


Fig. 3

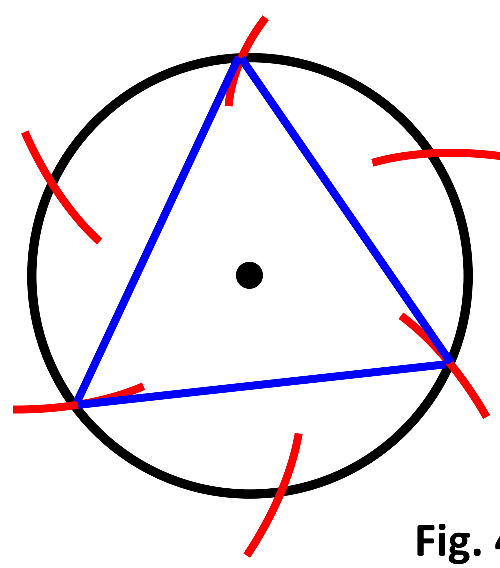


Fig. 4

Construct

a square inscribed in a circle

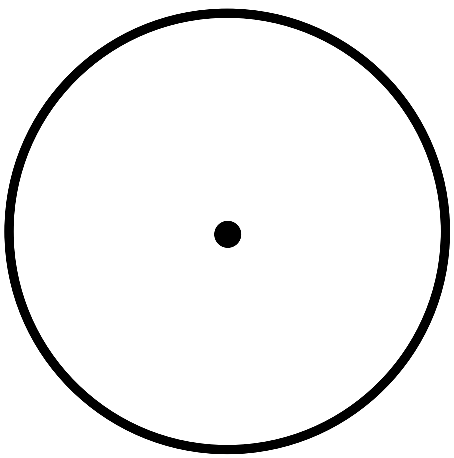


Fig. 1 **Draw a diameter.**

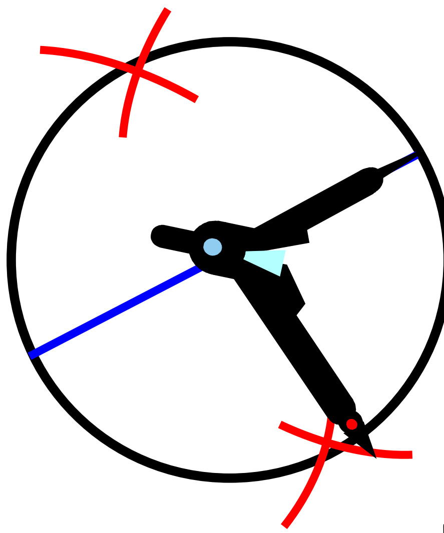


Fig. 2

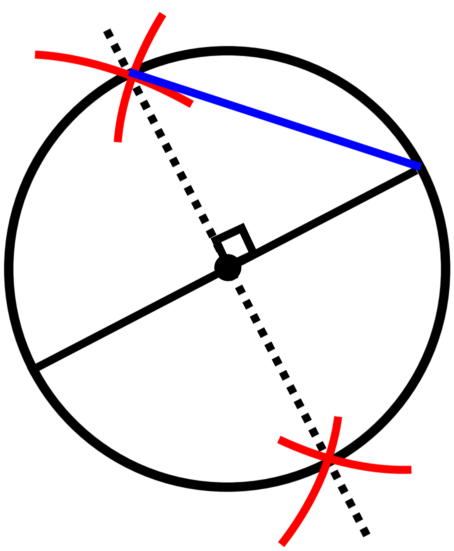


Fig. 3

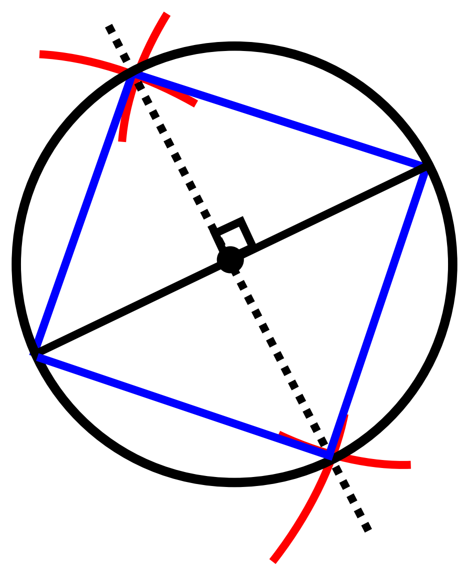


Fig. 4

Construct

a regular hexagon inscribed in a circle

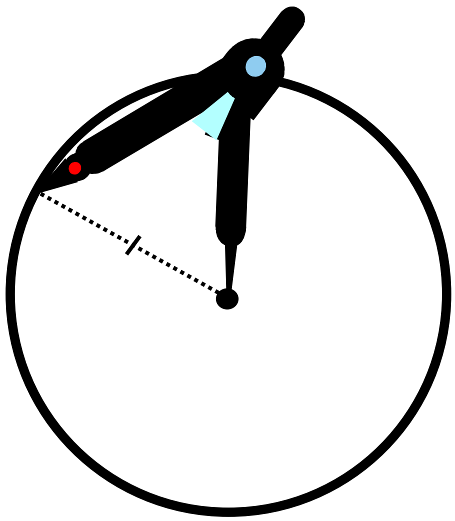


Fig. 1

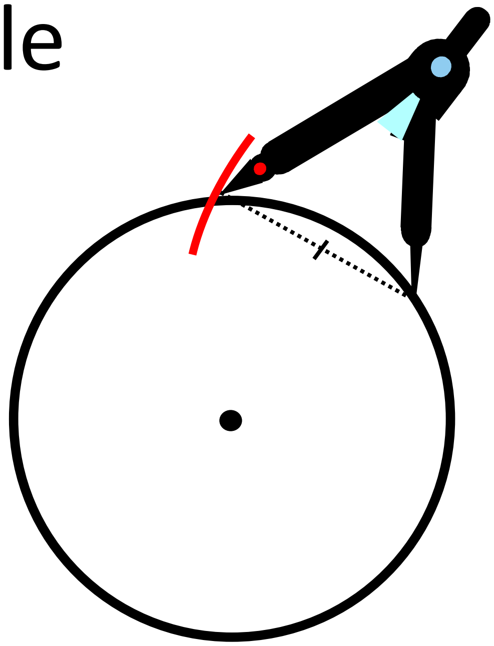


Fig. 2

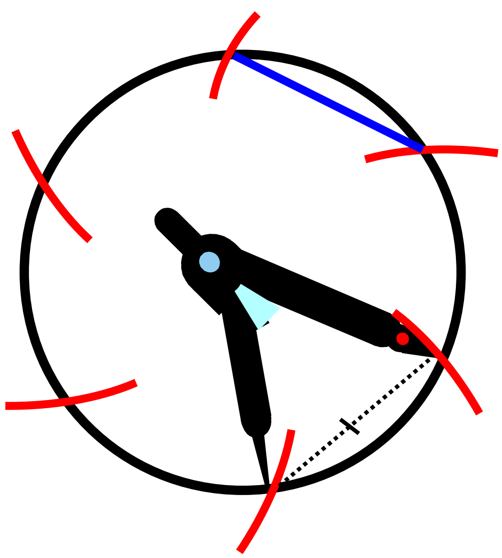


Fig. 3

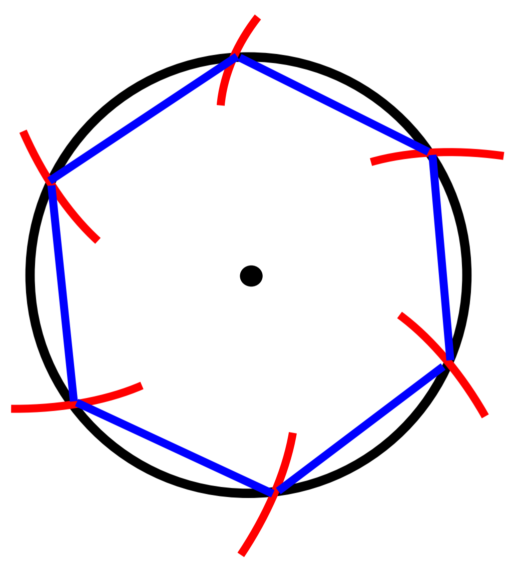


Fig. 4

Construct

the inscribed circle of a triangle

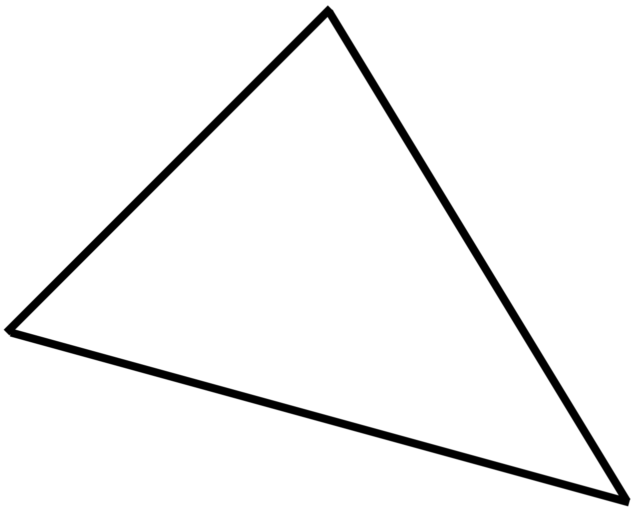


Fig. 1

Bisect all angles.

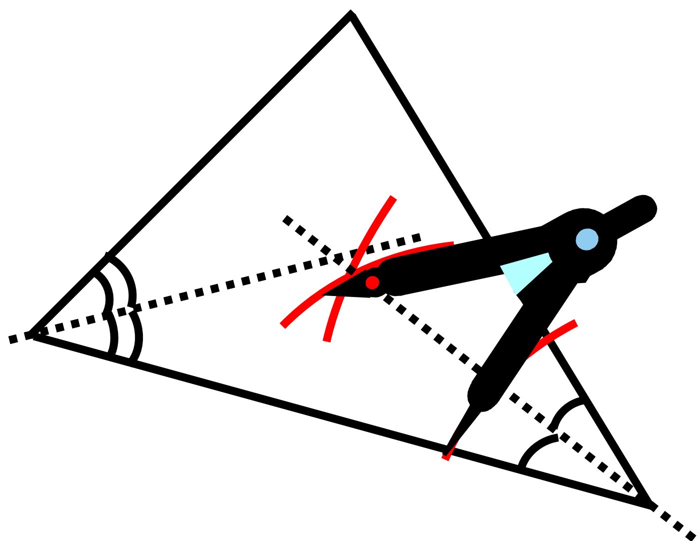


Fig. 2

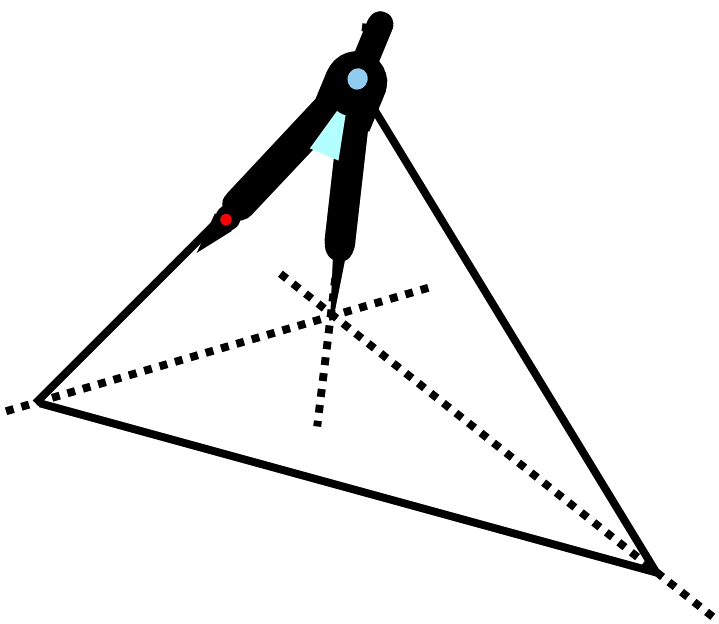


Fig. 3

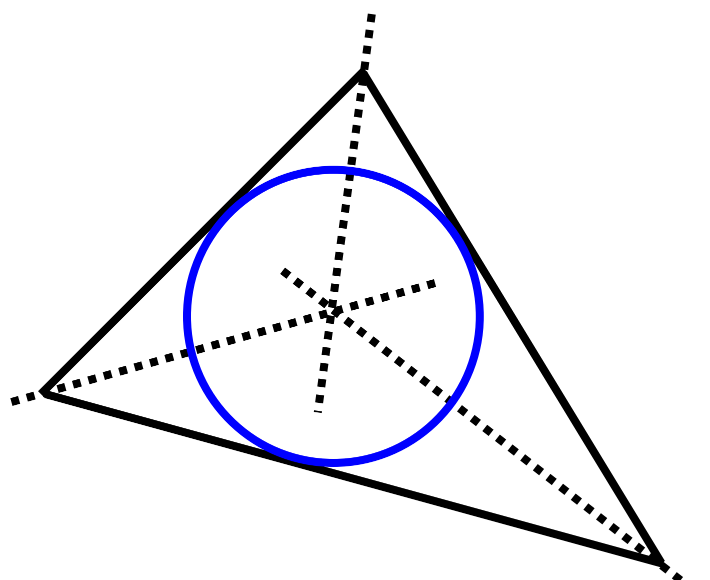


Fig. 4

Construct

the circumscribed circle of a triangle

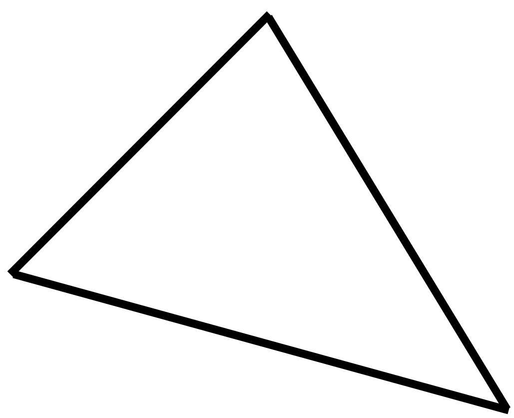


Fig. 1

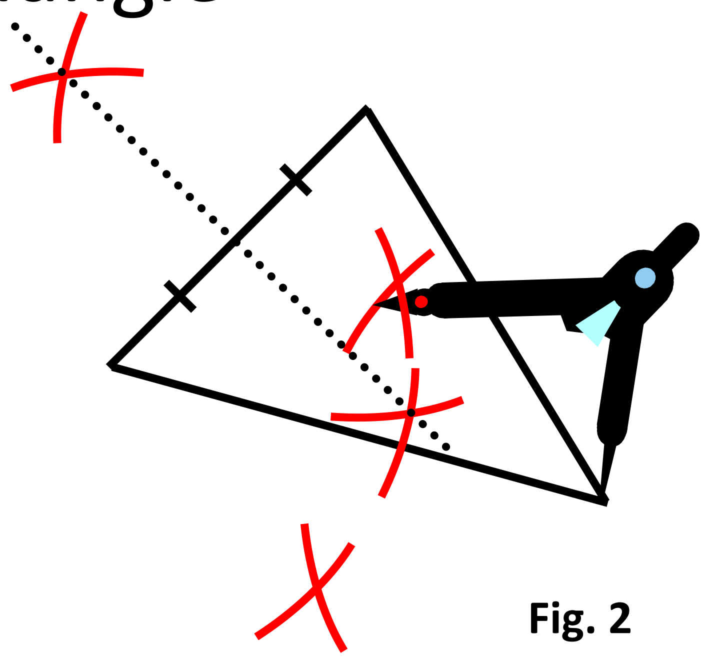


Fig. 2

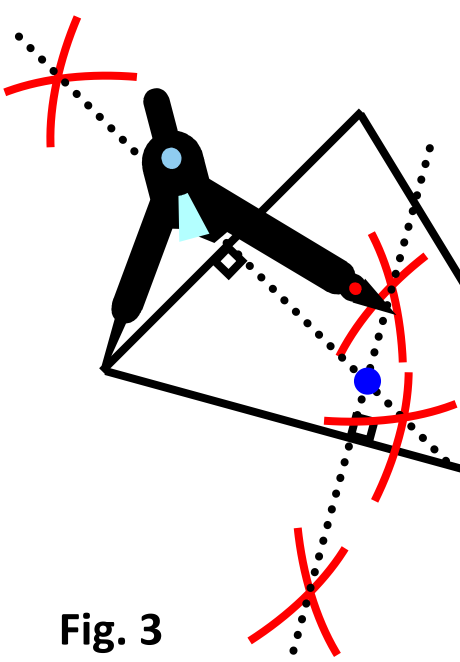


Fig. 3

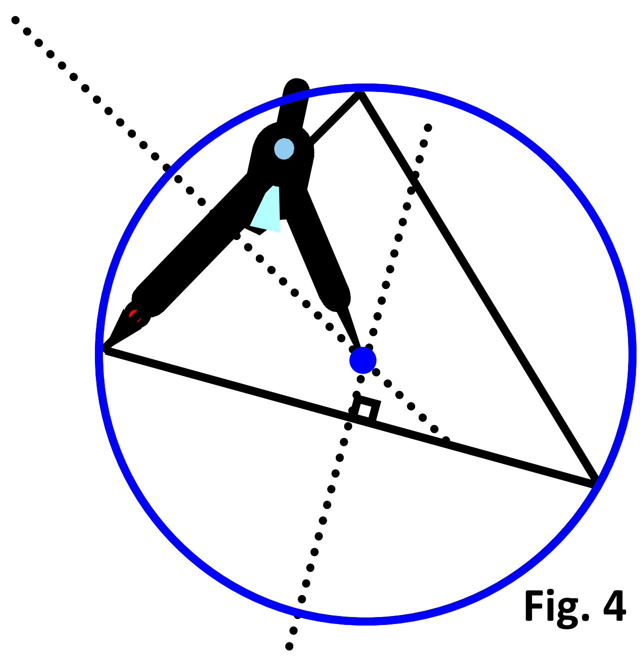


Fig. 4

Construct

a tangent from a point outside a given circle to the circle

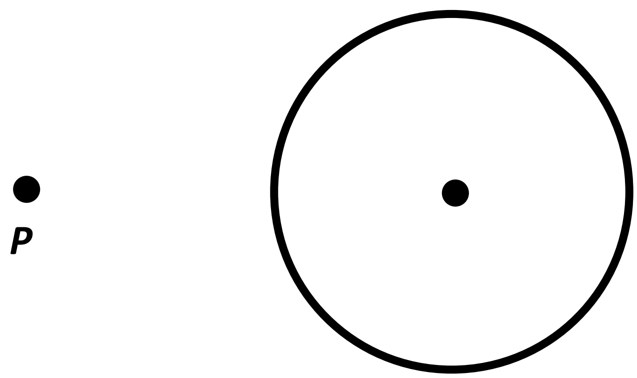


Fig. 1

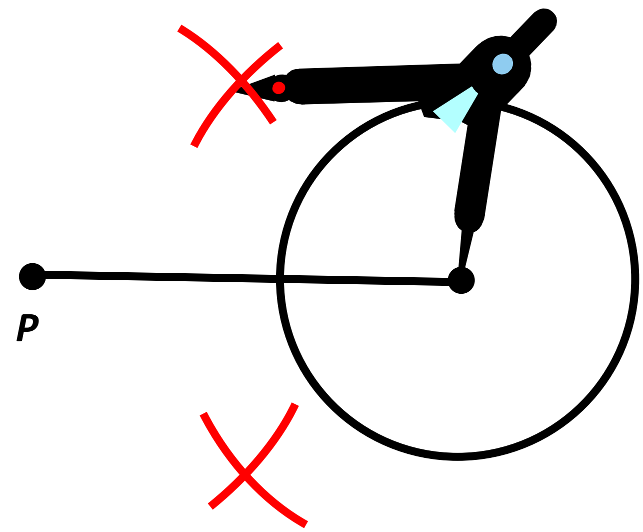


Fig. 2

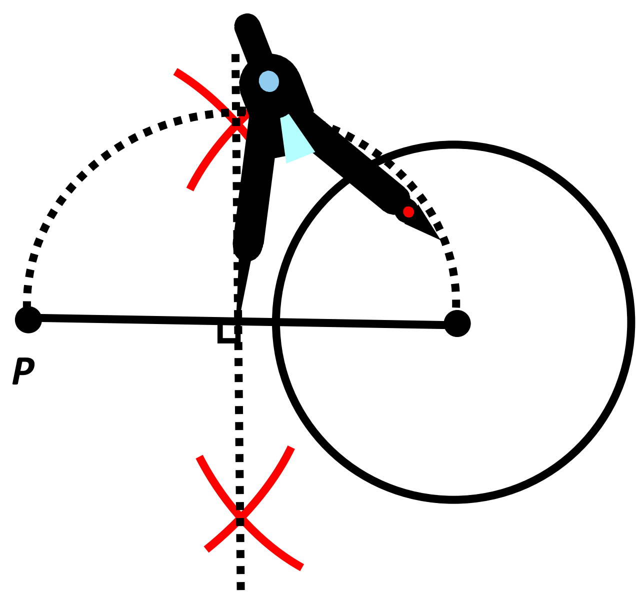


Fig. 3

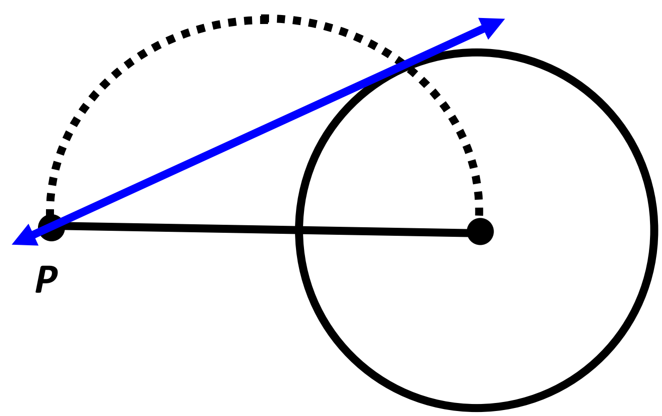
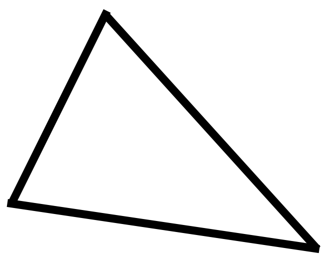
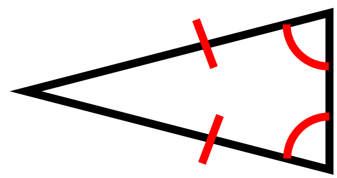
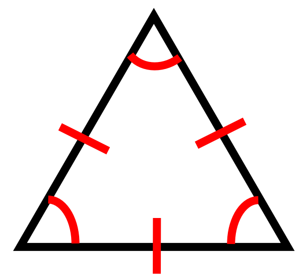


Fig. 4

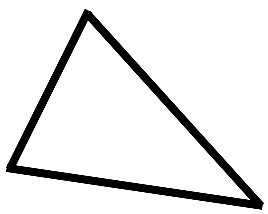
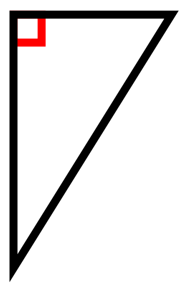
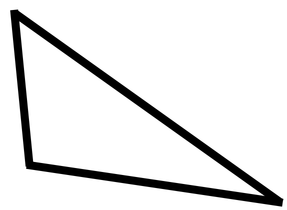
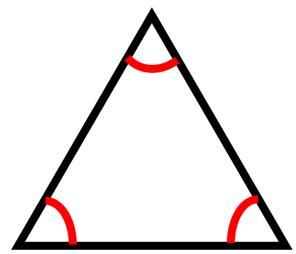
Triangles

Classifying Triangles

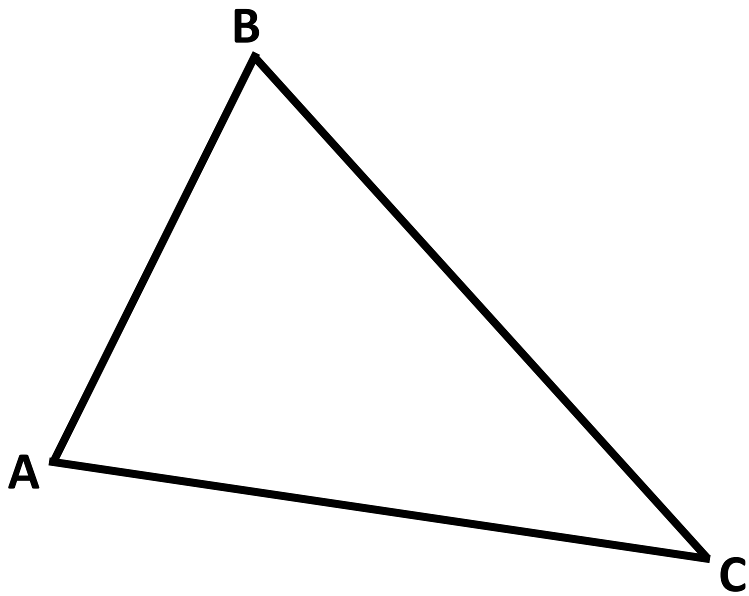
Scalene	***Isosceles***	***Equilateral***
		
No congruent sides	At least 2 congruent sides	3 congruent sides
No congruent angles	2 or 3 congruent angles	3 congruent angles

All equilateral triangles are isosceles.

Classifying Triangles

Acute	*Right*	*Obtuse*	*Equiangular*
			
3 acute angles	1 right angle	1 obtuse angle	3 congruent angles
3 angles, each less than 90°	1 angle equals 90°	1 angle greater than 90°	3 angles, each measures 60°

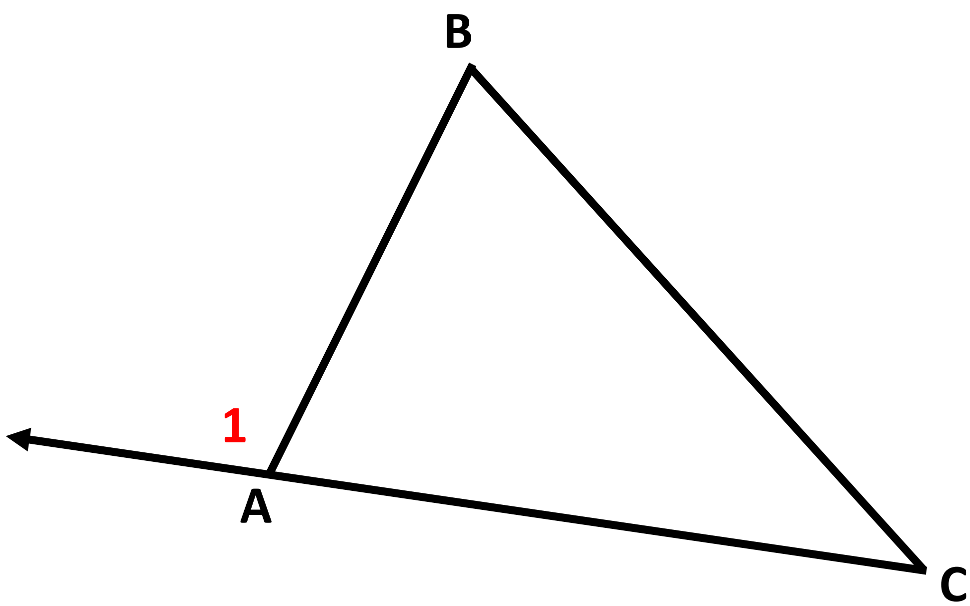
Triangle Sum Theorem



measures of the interior angles of a triangle = 180°

$m\angle A + m\angle B + m\angle C = 180°$

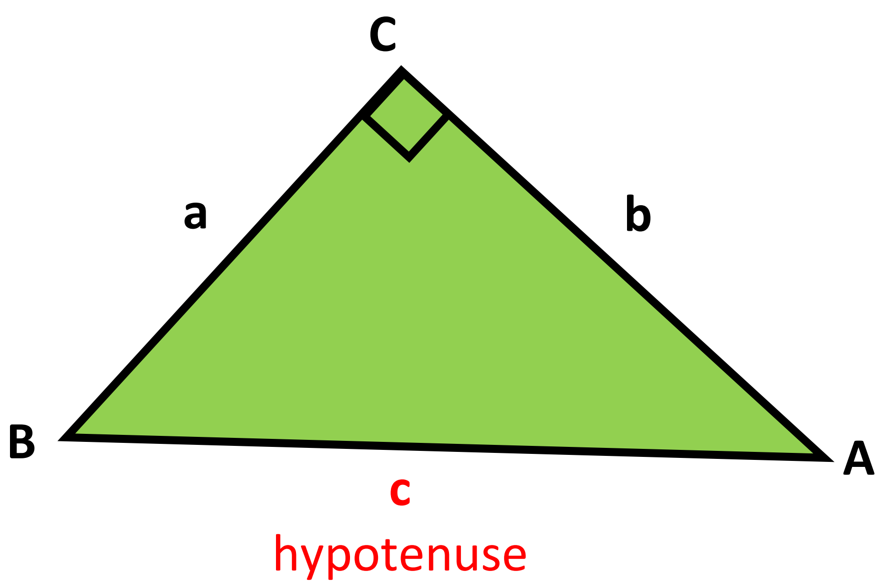
Exterior Angle Theorem



Exterior angle, $m\angle 1$, is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle B + m\angle C$$

Pythagorean Theorem

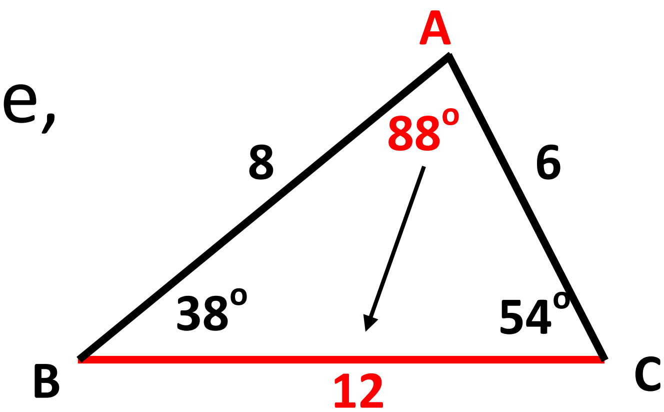


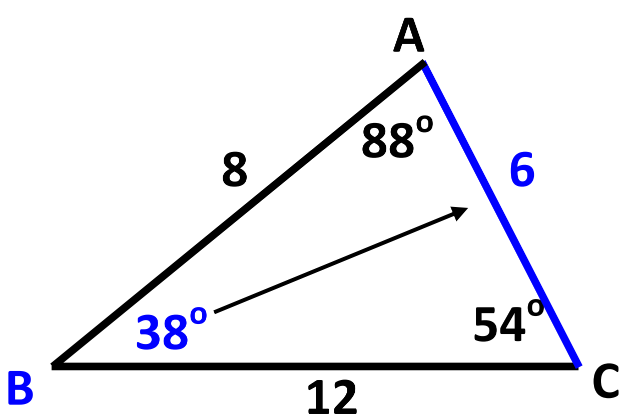
If ΔABC is a right triangle, then $a^2 + b^2 = c^2$.

Conversely, if $a^2 + b^2 = c^2$, then ΔABC is a right triangle.

Angle and Side Relationships

$\angle A$ is the largest angle, therefore $\overline{BC}$ is the longest side.

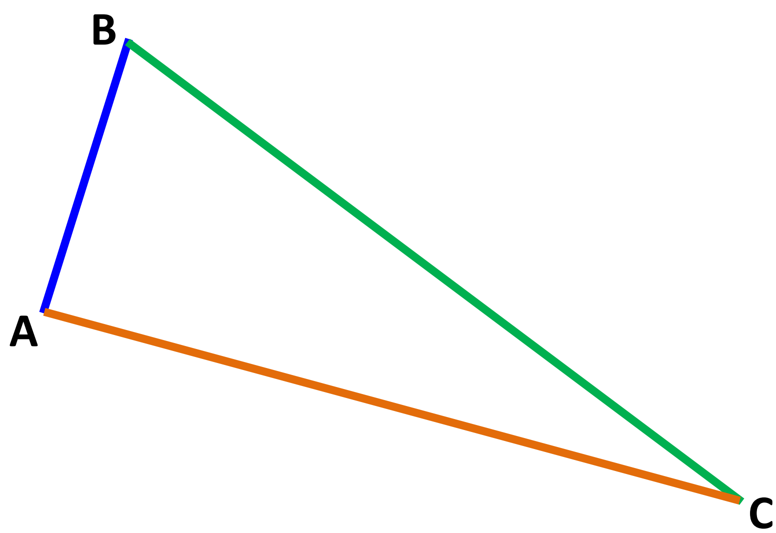




$\angle B$ is the smallest angle, therefore $\overline{AC}$ is the shortest side.

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

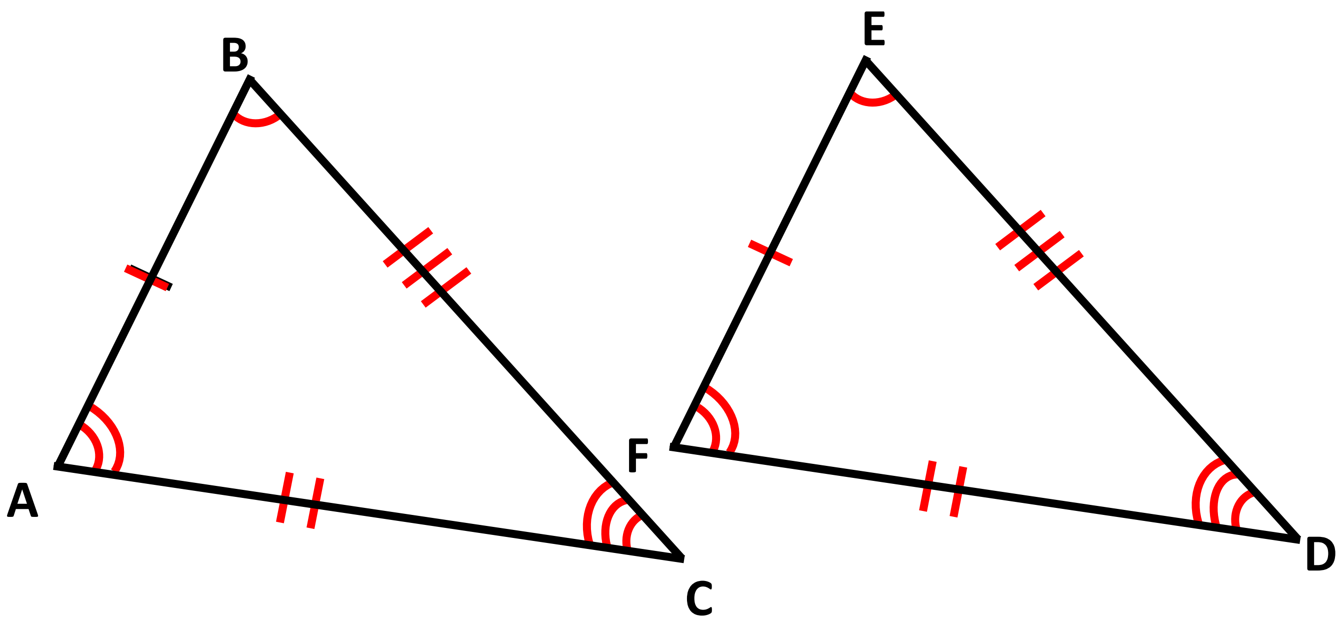


Example:

AB + BC > AC AC + BC > AB

AB + AC > BC

Congruent Triangles



Two possible congruence statements:

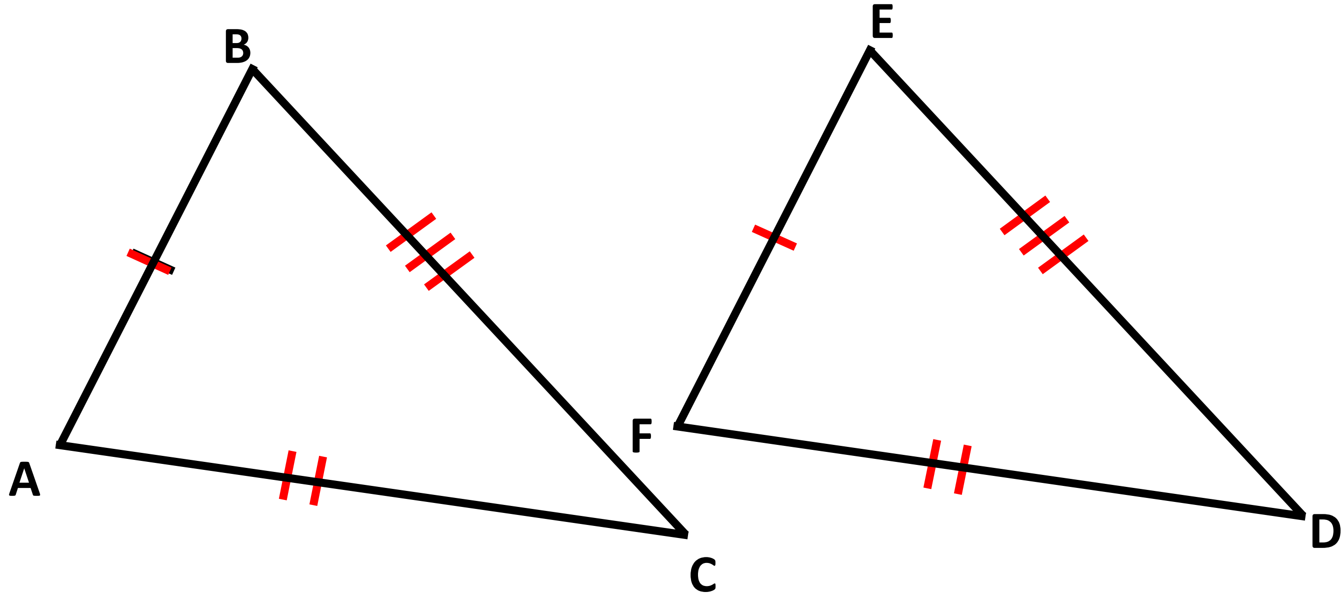
$\Delta ABC \cong \Delta FED$

$\Delta BCA \cong \Delta EDF$

Corresponding Parts of Congruent Figures

$\angle A \cong \angle F$	$AB \cong FE$
$\angle B \cong \angle E$	$BC \cong ED$
$\angle C \cong \angle D$	$CA \cong DF$

SSS Triangle Congruence Postulate



Example:

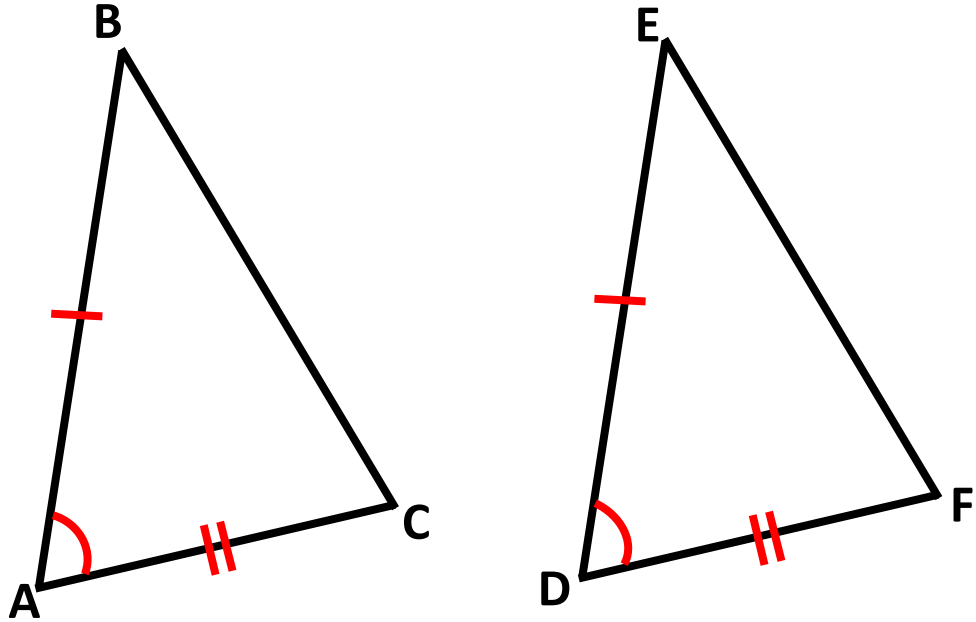
If Side $\overline{AB} \cong \overline{FE}$,

Side $\overline{AC} \cong \overline{FD}$, and

Side $\overline{BC} \cong \overline{ED}$,

then Δ ABC $\cong$ ΔFED.

SAS Triangle Congruence Postulate



Example:

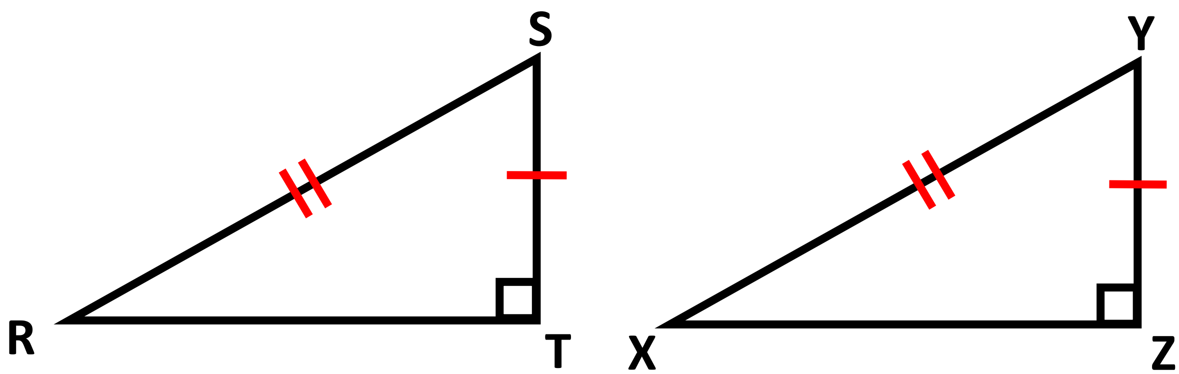
If Side $\overline{AB} \cong \overline{DE}$,

Angle $\angle A \cong \angle D$, and

Side $\overline{AC} \cong \overline{DF}$,

then $\Delta\ ABC \cong \Delta DEF$.

HL Right Triangle Congruence



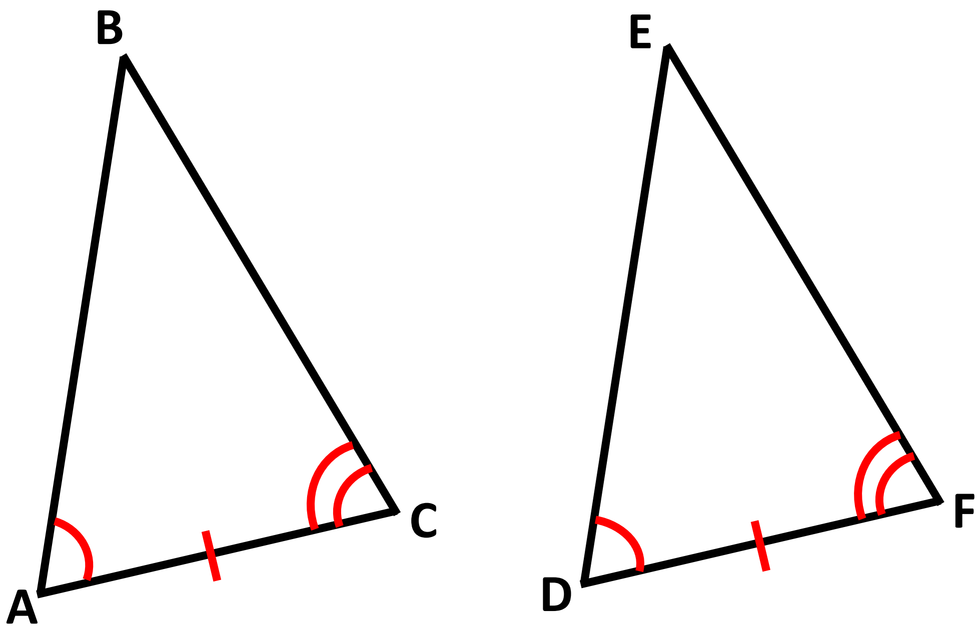
Example:

If Hypotenuse $\overline{RS} \cong \overline{XY}$, and

Leg $\overline{ST} \cong \overline{YZ}$,

then Δ RST $\cong$ ΔXYZ.

ASA Triangle Congruence Postulate



Example:

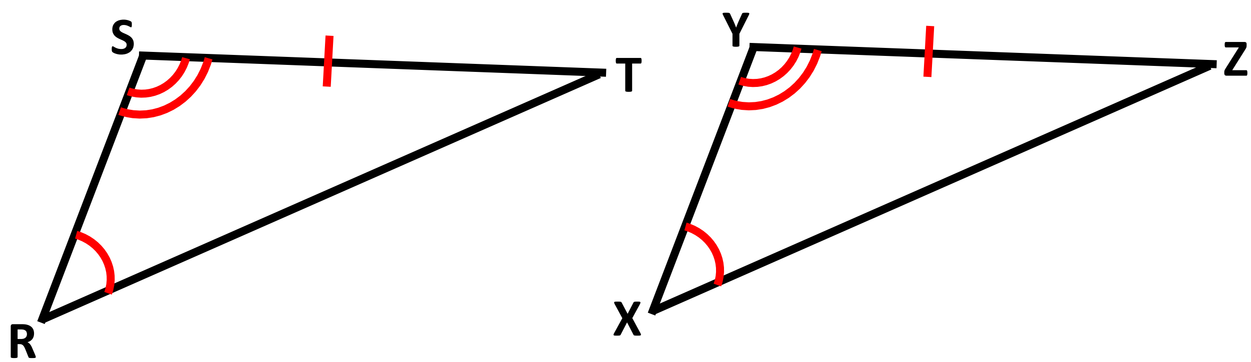
If Angle $\angle A \cong \angle D$,

Side $\overline{AC} \cong \overline{DF}$, and

Angle $\angle C \cong \angle F$

then $\Delta ABC \cong \Delta DEF$.

AAS Triangle Congruence Theorem



Example:

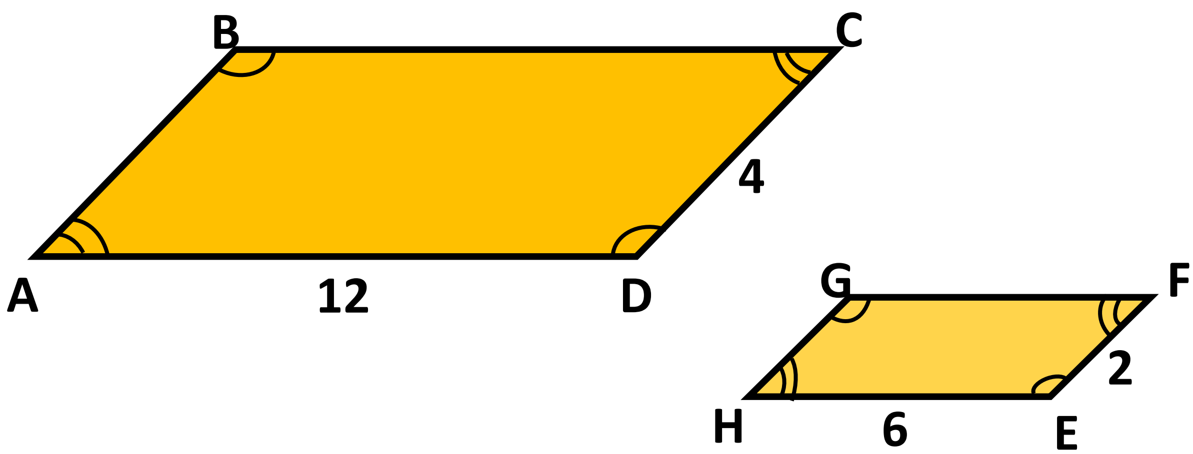
If Angle $\angle R \cong \angle X$,

Angle $\angle S \cong \angle Y$, and

Side $\overline{ST} \cong \overline{YZ}$

then Δ RST $\cong$ ΔXYZ.

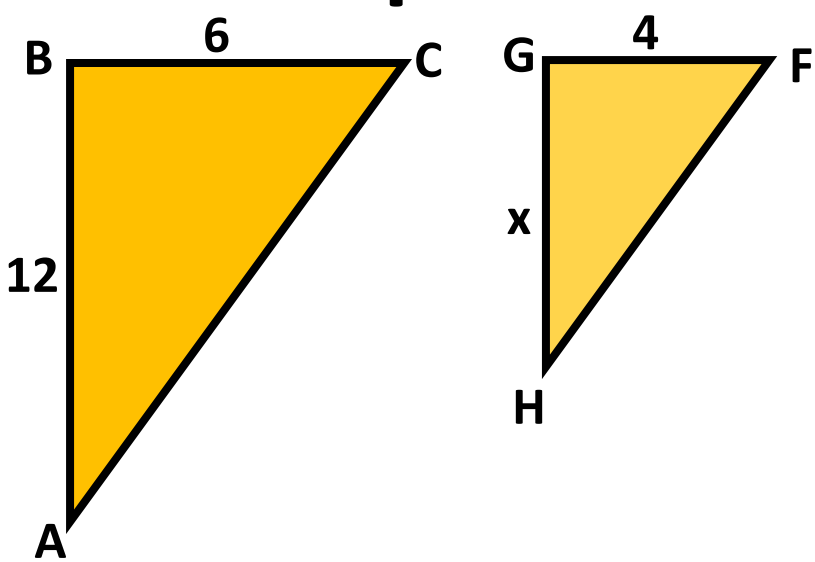
Similar Polygons



ABCD ~ HGFE	
Angles	**Sides**
$\angle A$ corresponds to $\angle H$	$\overline{AB}$ corresponds to $\overline{HG}$
$\angle B$ corresponds to $\angle G$	$\overline{BC}$ corresponds to $\overline{GF}$
$\angle C$ corresponds to $\angle F$	$\overline{CD}$ corresponds to $\overline{FE}$
$\angle D$ corresponds to $\angle E$	$\overline{DA}$ corresponds to $\overline{EH}$

Corresponding angles are congruent.
Corresponding sides are proportional.

Similar Polygons and Proportions



Corresponding vertices are listed in the same order.

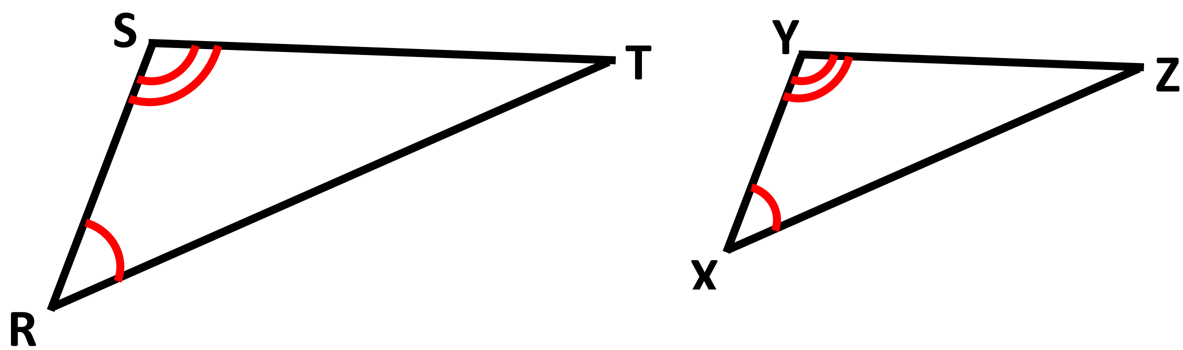
Example: $\Delta ABC \sim \Delta HGF$

$$\frac{AB}{HG} = \frac{BC}{GF}$$

$$\frac{12}{x} = \frac{6}{4}$$

The perimeters of the polygons are also proportional.

AA Triangle Similarity Postulate



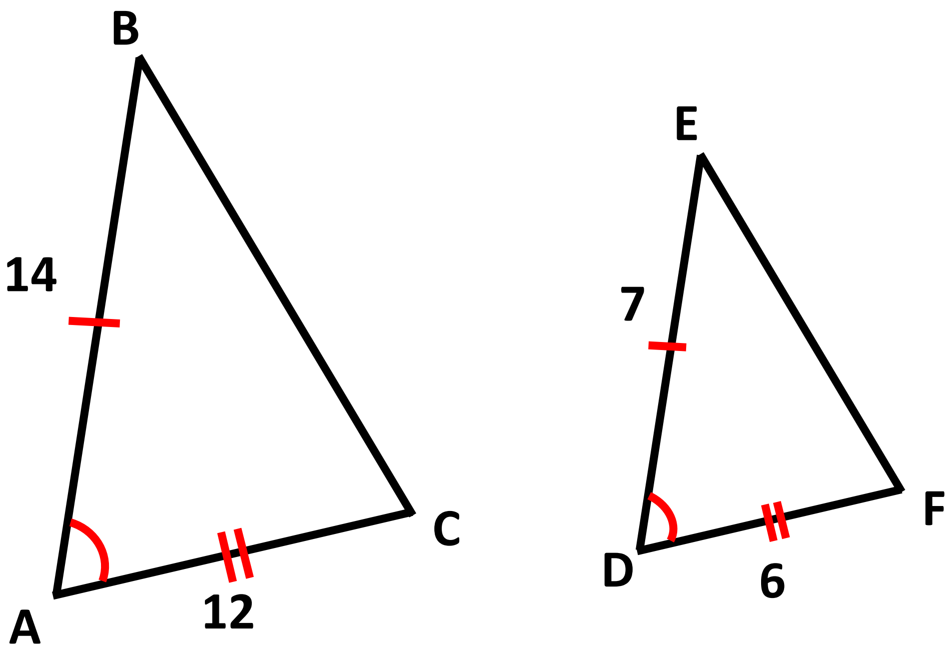
Example:

If Angle $\angle R \cong \angle X$ and

Angle $\angle S \cong \angle Y$,

then $\Delta RST \sim \Delta XYZ$.

SAS Triangle Similarity Theorem



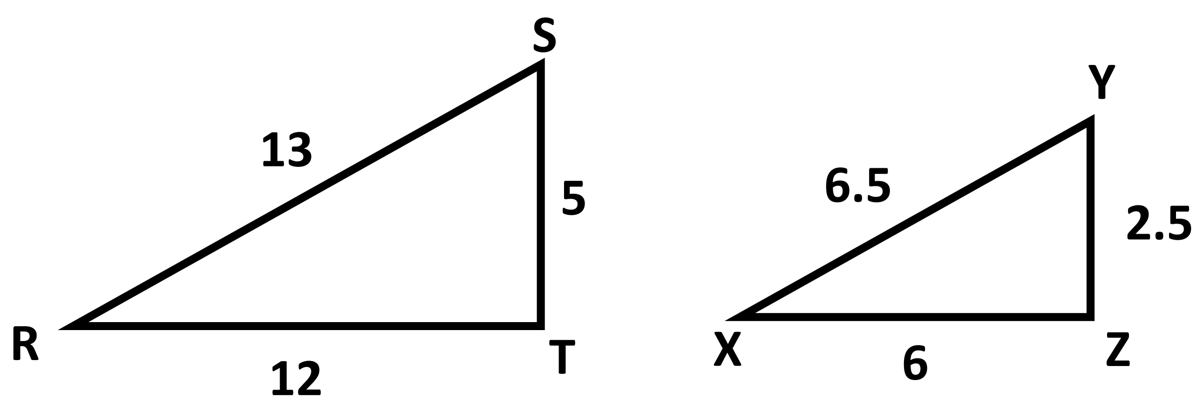
Example:

If $\angle A \cong \angle D$ and

$$\frac{AB}{DE} = \frac{AC}{DF}$$

then $\Delta ABC \sim \Delta DEF$.

SSS Triangle Similarity Theorem



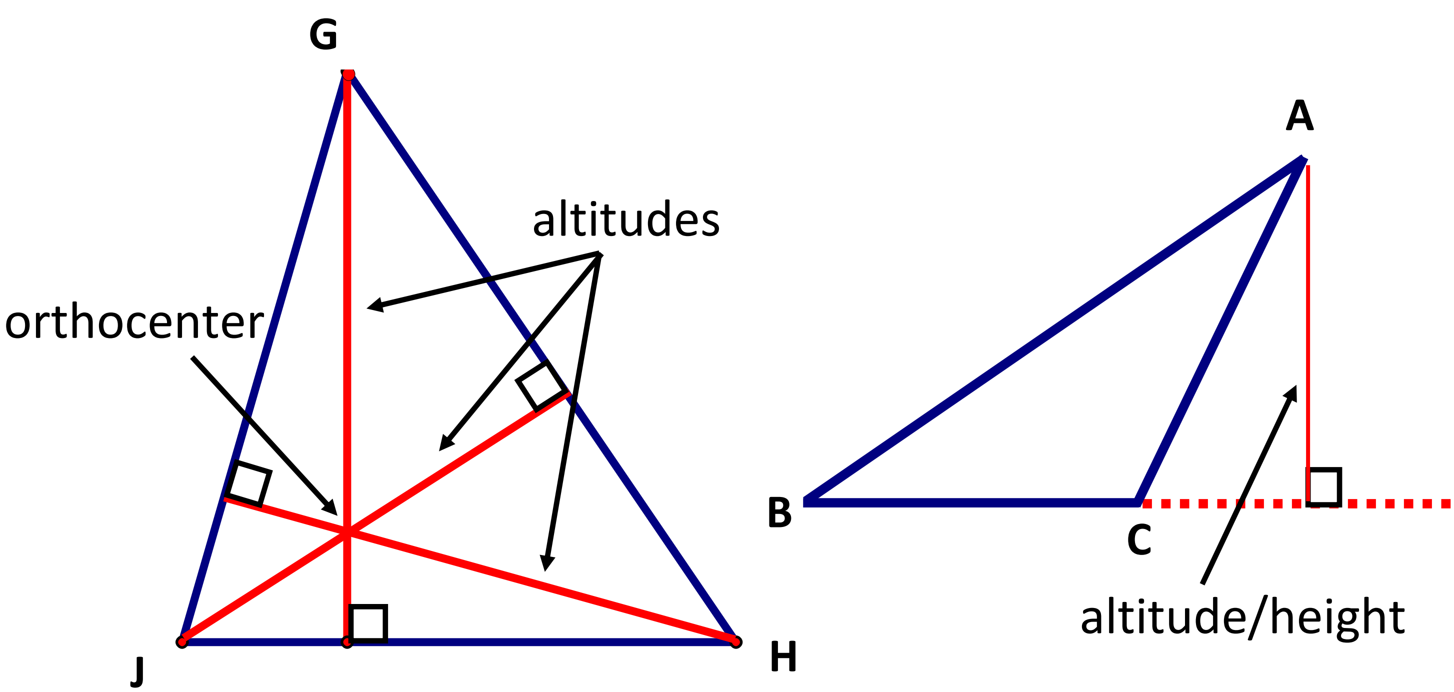
Example:

$$\text{If } \frac{RT}{XZ} = \frac{RS}{XY} = \frac{ST}{YZ}$$

then $\Delta RST \sim \Delta XYZ$.

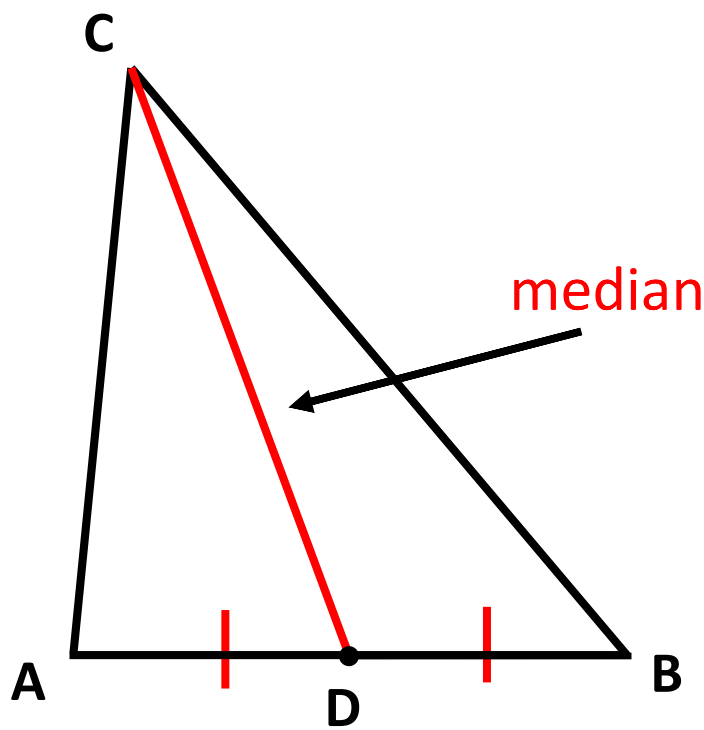
Altitude of a Triangle

a segment from a vertex perpendicular to the opposite side



Every triangle has 3 altitudes.
The 3 altitudes intersect at a point called the orthocenter.

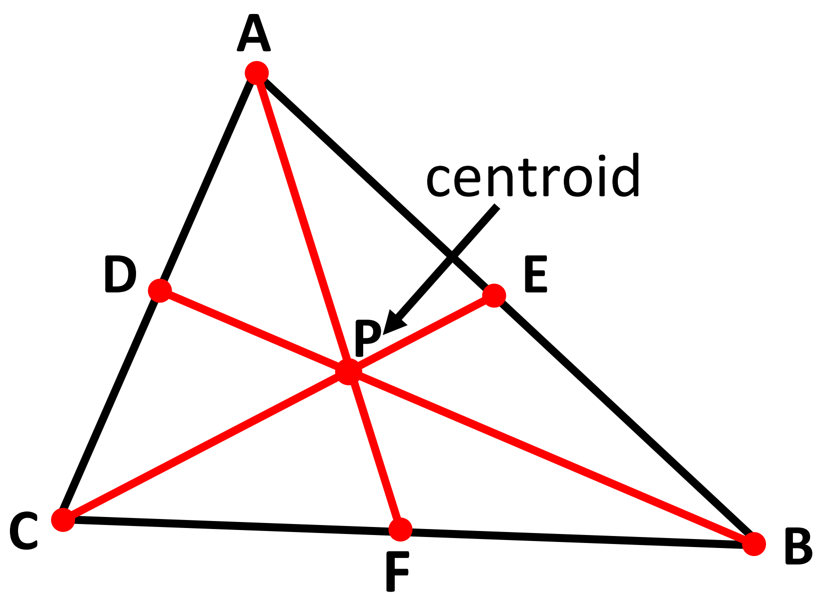
Median of a Triangle



D is the midpoint of $\overline{AB}$; therefore, $\overline{CD}$ is a median of ΔABC.

Every triangle has 3 medians.

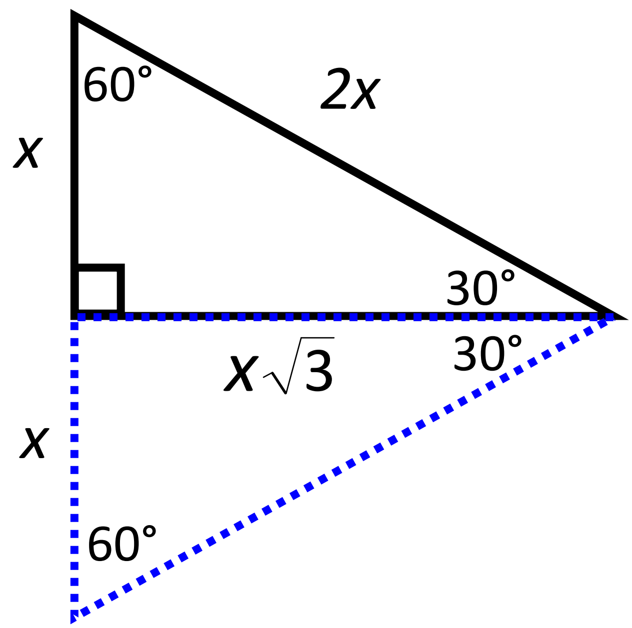
Concurrency of Medians of a Triangle



Medians of ΔABC intersect at P and

$$AP = \frac{2}{3}AF, \quad CP = \frac{2}{3}CE, \quad BP = \frac{2}{3}BD.$$

30°-60°-90° Triangle Theorem



Given: short leg = x

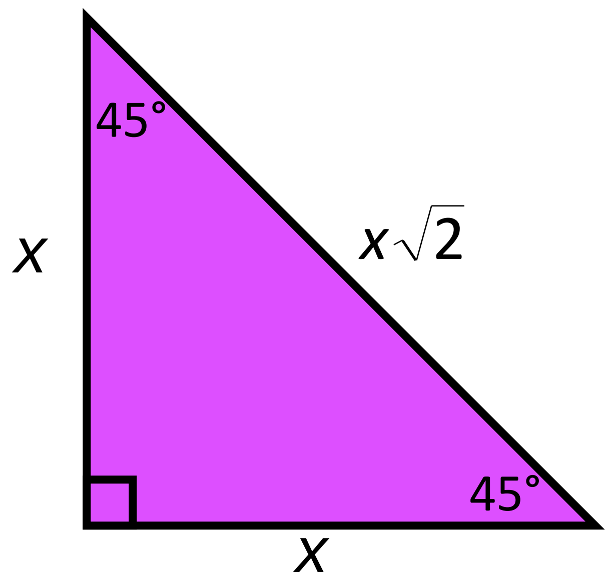
Using equilateral triangle,

hypotenuse = $2 \cdot x$

Applying the Pythagorean Theorem,

longer leg = $x \cdot \sqrt{3}$

45°-45°-90° Triangle Theorem



Given: leg = *x*,

then applying the Pythagorean Theorem;

$$\text{hypotenuse}^2 = x^2 + x^2$$

$$\text{hypotenuse} = x\sqrt{2}$$

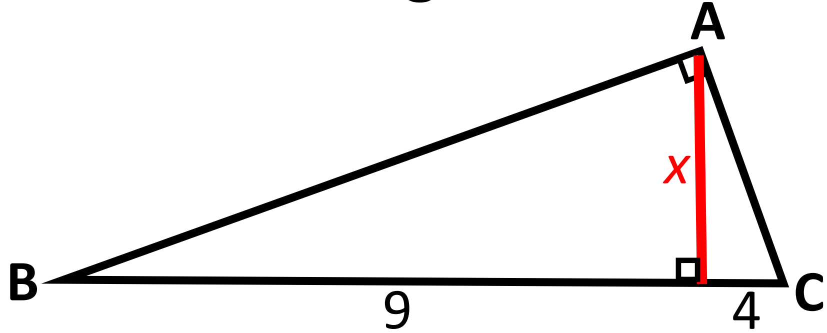
Geometric Mean

of two positive numbers *a* and *b* is the positive number *x* that satisfies

$$\frac{a}{x} = \frac{x}{b}.$$

$$x^2 = ab \text{ and } x = \sqrt{ab}.$$

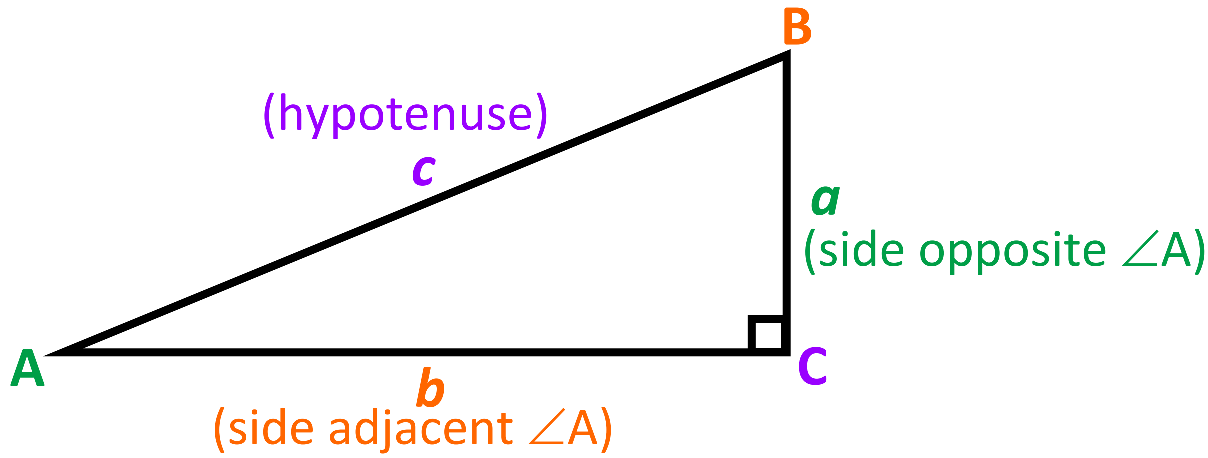
In a right triangle, the length of the altitude is the geometric mean of the lengths of the two segments.



Example:

$\frac{9}{x} = \frac{x}{4}$, so $x^2 = 36$ and $x = \sqrt{36} = 6$.

Trigonometric Ratios

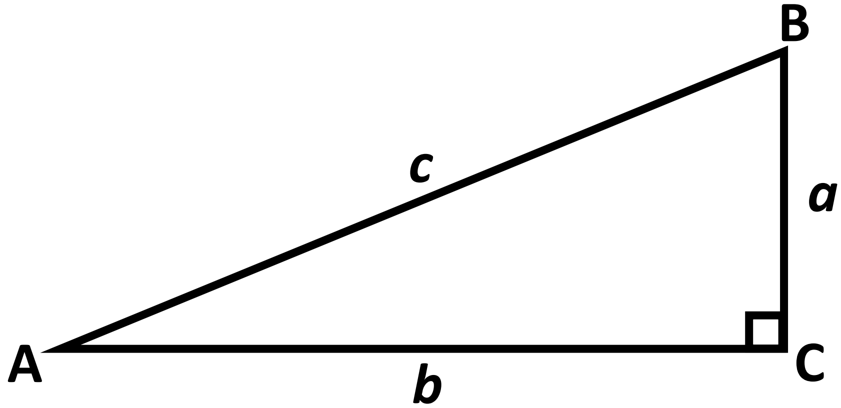


$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

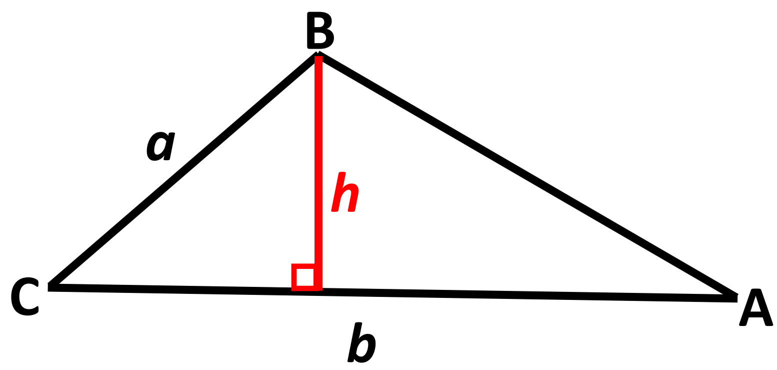
$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$

Inverse Trigonometric Ratios



Definition	Example
If tan A = x, then $\tan^{-1} x = m\angle A$.	$\tan^{-1} \frac{a}{b} = m\angle A$
If sin A = y, then $\sin^{-1} y = m\angle A$.	$\sin^{-1} \frac{a}{c} = m\angle A$
If cos A = z, then $\cos^{-1} z = m\angle A$.	$\cos^{-1} \frac{b}{c} = m\angle A$

Area of Triangle



$$\sin C = \frac{h}{a}$$

$$h = a \cdot \sin C$$

$$A = \frac{1}{2}bh$$ (area of a triangle formula)

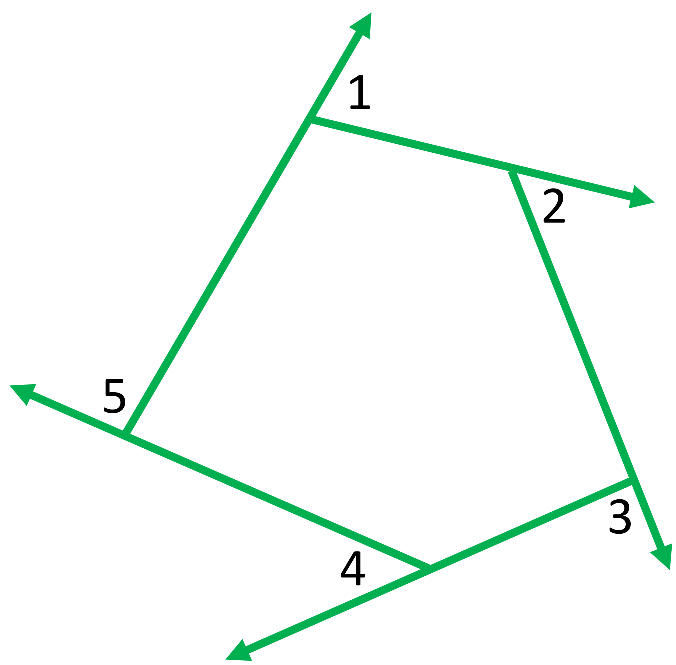
By substitution, $A = \frac{1}{2}b(a \cdot \sin C)$

$$A = \frac{1}{2}ab \cdot \sin C$$

Polygons and Circles

Polygon Exterior Angle Sum Theorem

The sum of the measures of the exterior angles of a convex polygon is 360°.



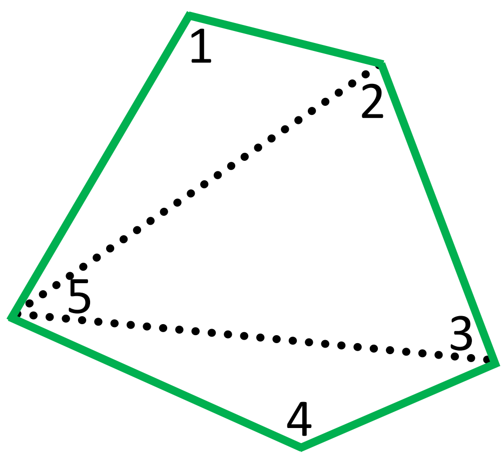
Example:

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360^\circ$

Polygon Interior Angle Sum Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

$$S = m\angle 1 + m\angle 2 + \ldots + m\angle n = (n - 2) \cdot 180°$$



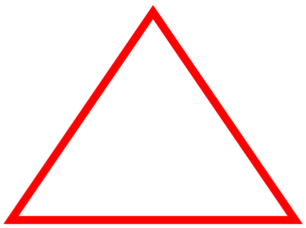
Example:

If $n = 5$, then $S = (5 - 2) \cdot 180°$

$S = 3 \cdot 180° = 540°$

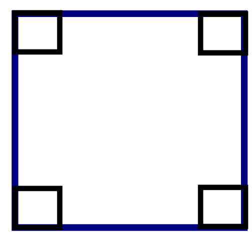
Regular Polygon

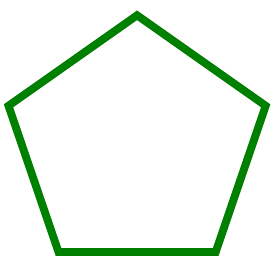
a convex polygon that is both equiangular and equilateral



Equilateral Triangle
Each angle measures 60°.

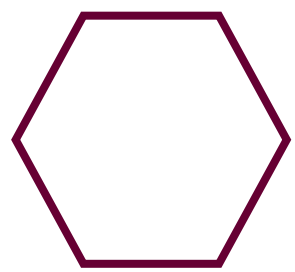
Square
Each angle measures 90°.

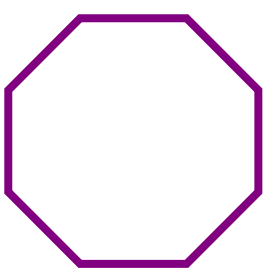




Regular Pentagon
Each angle measures 108°.

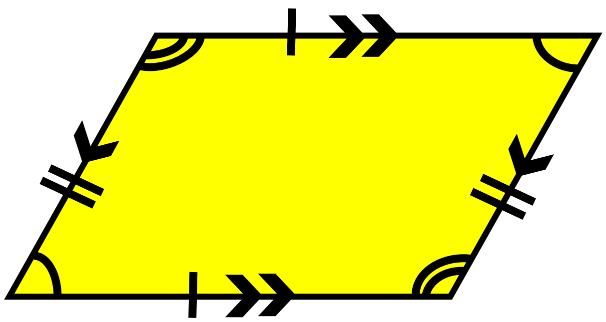
Regular Hexagon
Each angle measures 120°.



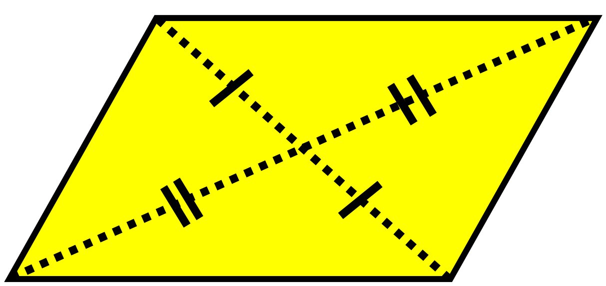


Regular Octagon
Each angle measures 135°.

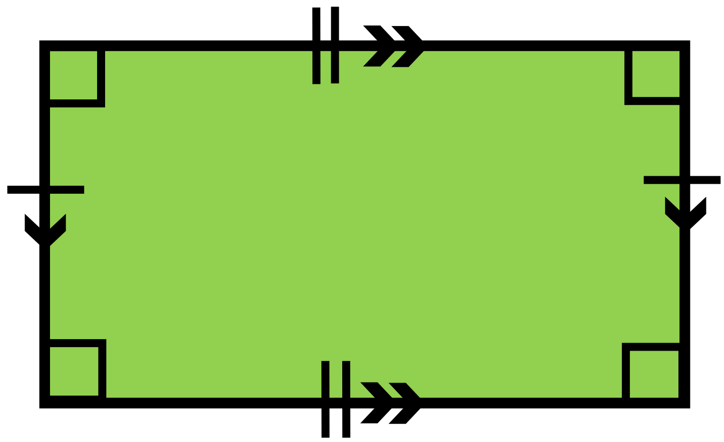
Properties of Parallelograms



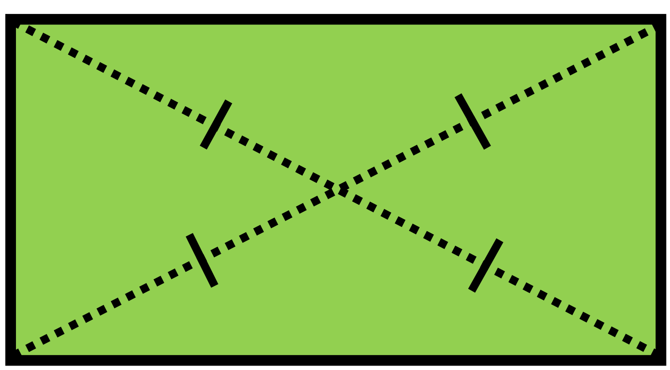
- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.



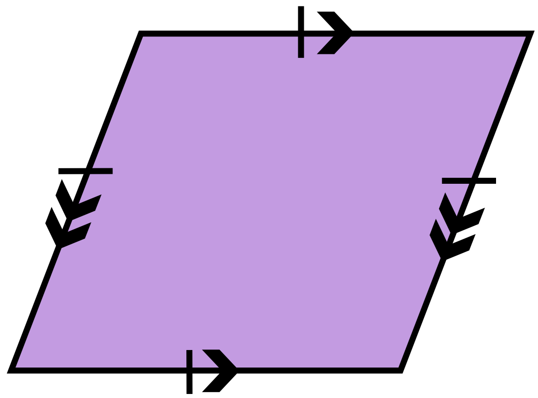
Rectangle



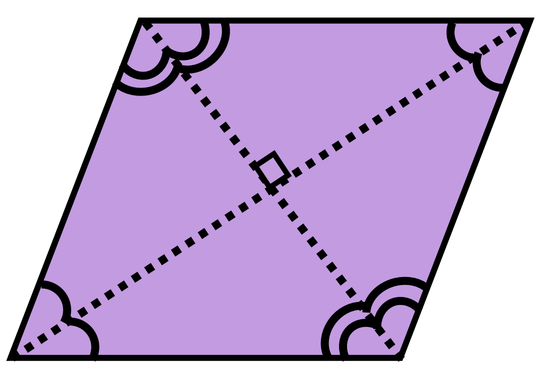
- A rectangle is a parallelogram with four right angles.
- Diagonals are congruent.
- Diagonals bisect each other.



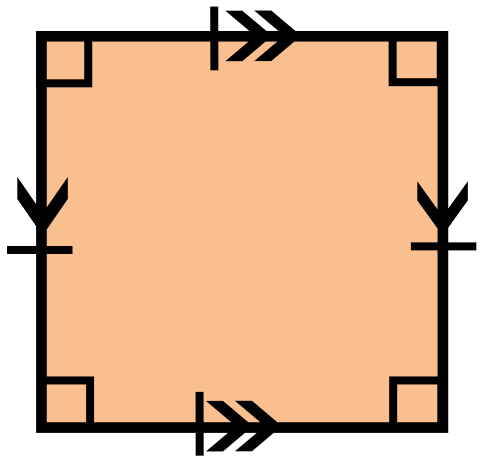
Rhombus



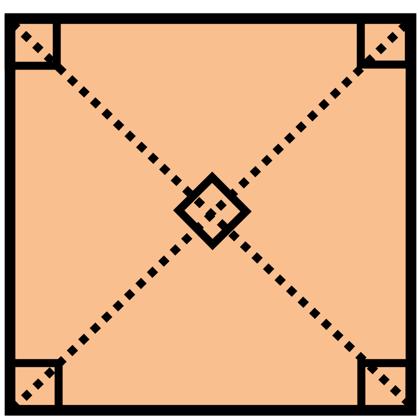
- A rhombus is a parallelogram with four congruent sides.
- Diagonals are perpendicular.
- Each diagonal bisects a pair of opposite angles.



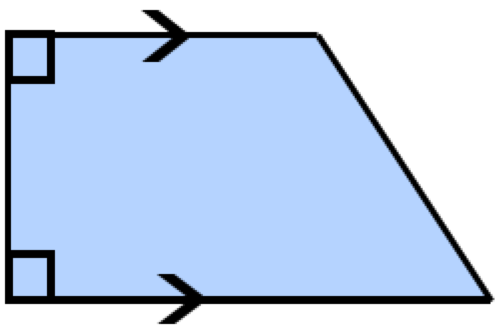
Square

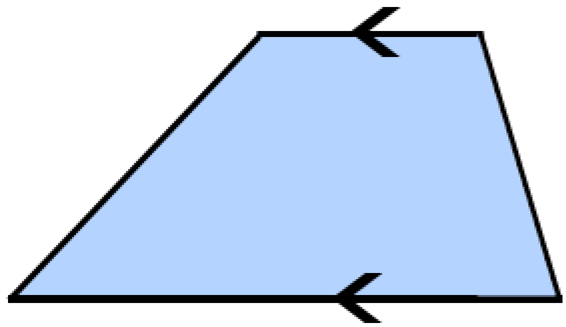


- A square is a parallelogram and a rectangle with four congruent sides.
- Diagonals are perpendicular.
- Every square is a rhombus.

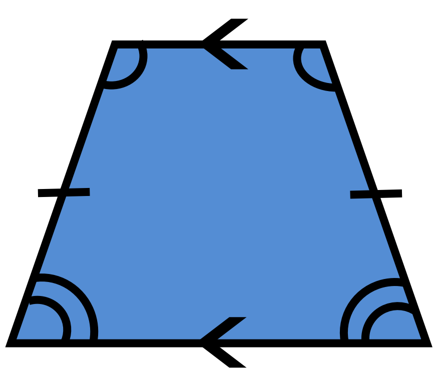


Trapezoid



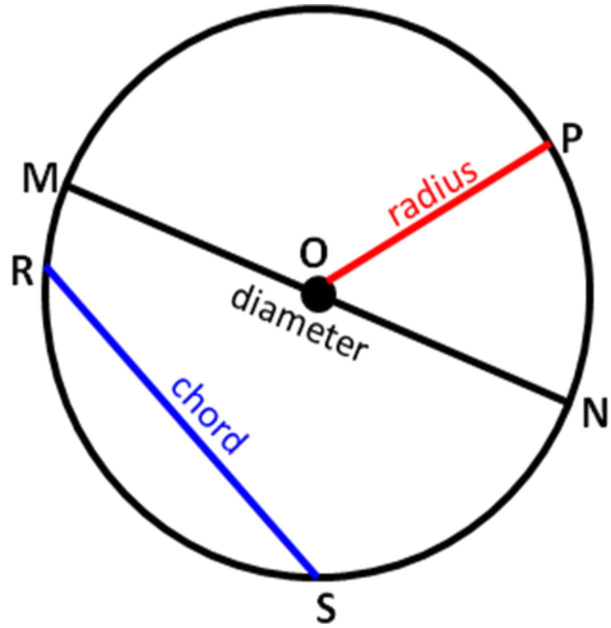


- A trapezoid is a quadrilateral with exactly one pair of parallel sides.
- Isosceles trapezoid – A trapezoid where the two base angles are equal and therefore the sides opposite the base angles are also equal.



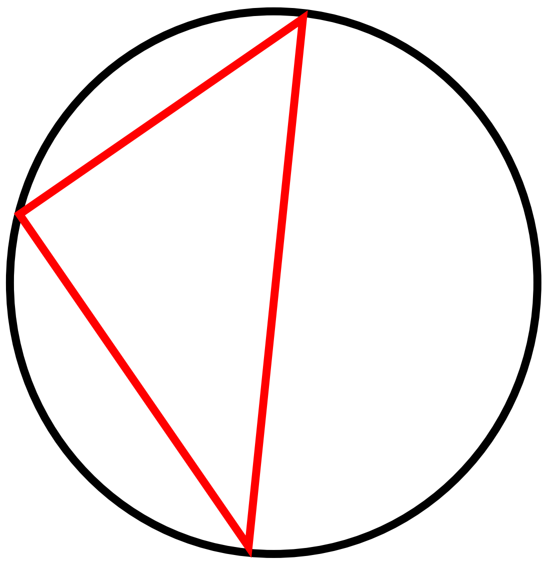
Circle

all points in a plane equidistant from a given point called the center



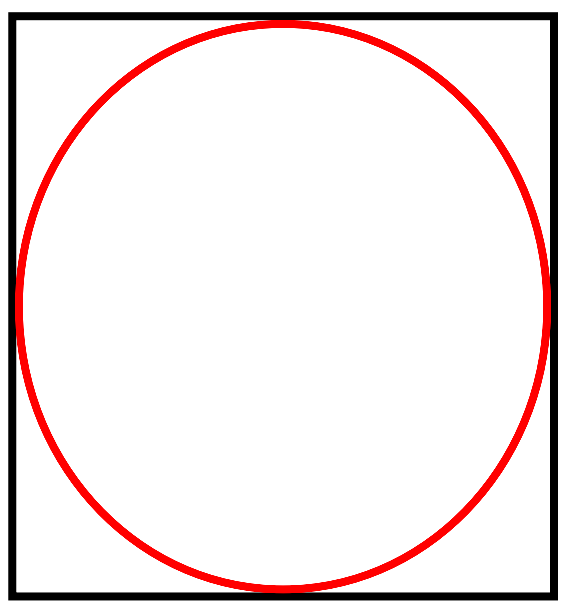
- Point O is the center.
- $\overline{MN}$ passes through the center O and therefore, $\overline{MN}$ is a diameter.
- $\overline{OP}$, $\overline{OM}$, and $\overline{ON}$ are radii and $\overline{OP} \cong \overline{OM} \cong \overline{ON}$.
- $\overline{RS}$ and $\overline{MN}$ are chords.

Circles

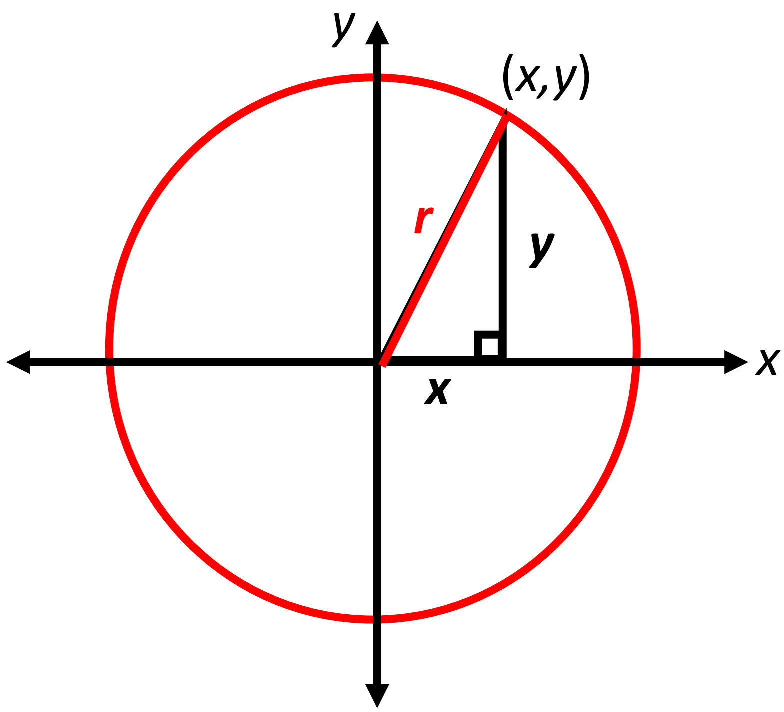


A polygon is an inscribed polygon if all of its vertices lie on a circle.

A circle is considered “inscribed” if it is tangent to each side of the polygon.



Circle Equation



$$x^2 + y^2 = r^2$$

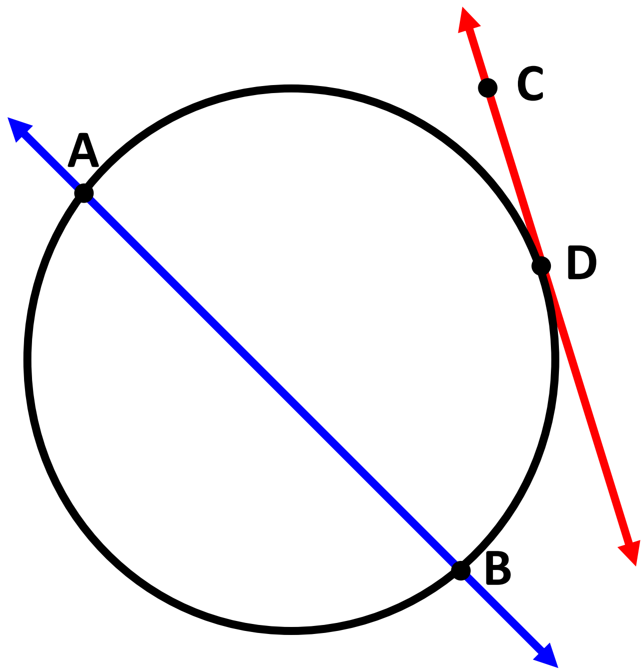
circle with radius r and center at the origin

standard equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

with center (h,k) and radius r

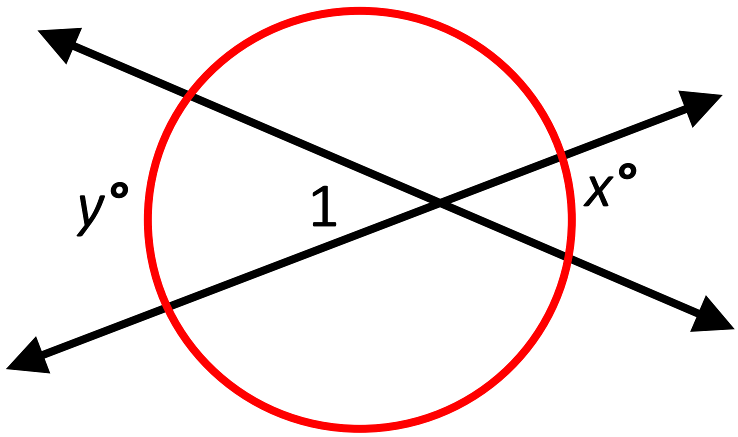
Lines and Circles



- Secant ($\overleftrightarrow{AB}$) – a line that intersects a circle in two points.

- Tangent ($\overleftrightarrow{CD}$) – a line (or ray or segment) that intersects a circle in exactly one point, the point of tangency, D.

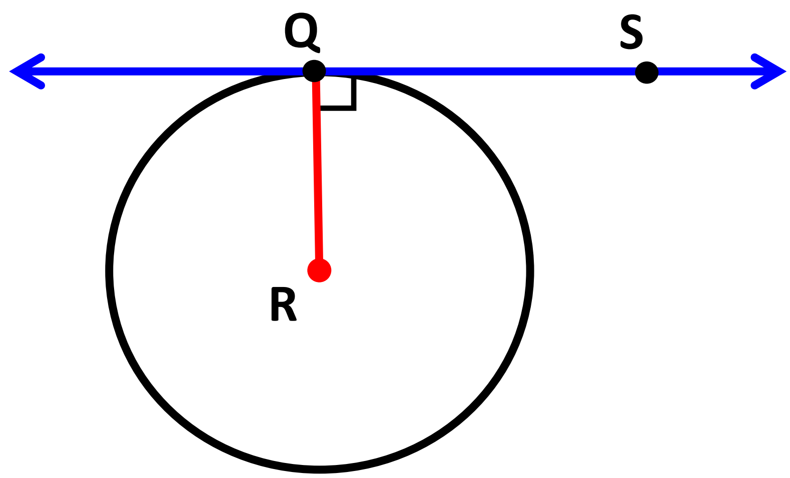
Secant



If two lines intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measures of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}(x^\circ + y^\circ)$$

Tangent

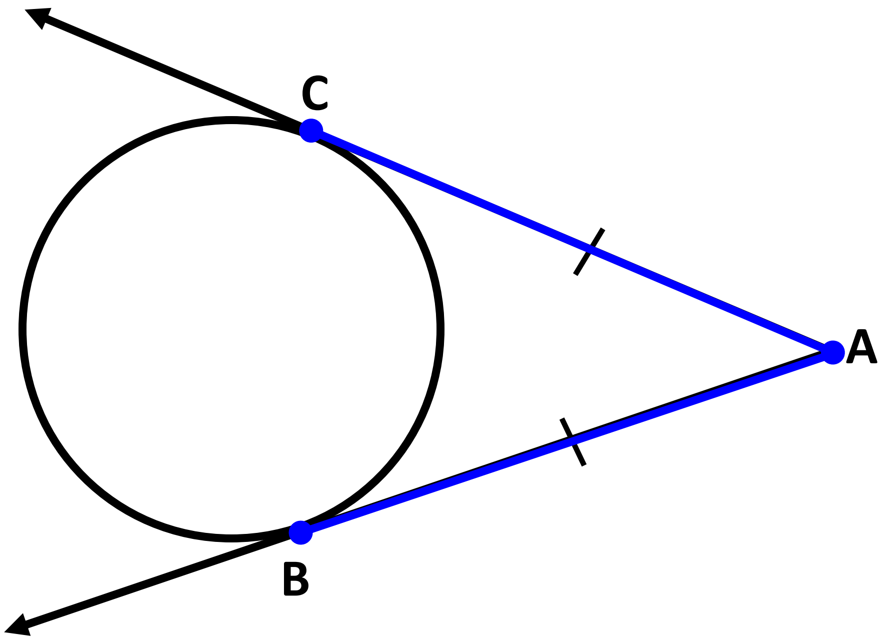


A line is tangent to a circle if and only if the line is perpendicular to a radius drawn to the point of tangency.

$\overleftrightarrow{QS}$ is tangent to circle R at point Q.

Radius $\overline{RQ} \perp \overleftrightarrow{QS}$

Tangent



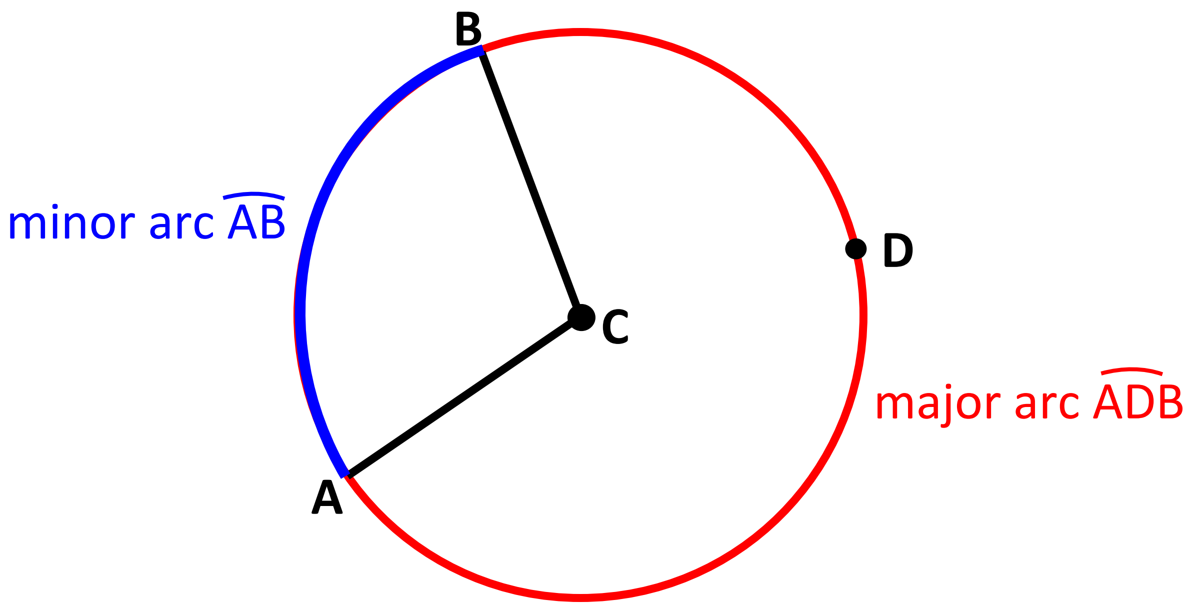
If two segments from the same exterior point are tangent to a circle, then they are congruent.

$\overline{AB}$ and $\overline{AC}$ are tangent to the circle at points B and C.

Therefore, $\overline{AB} \cong \overline{AC}$ and $AC = AB$.

Central Angle

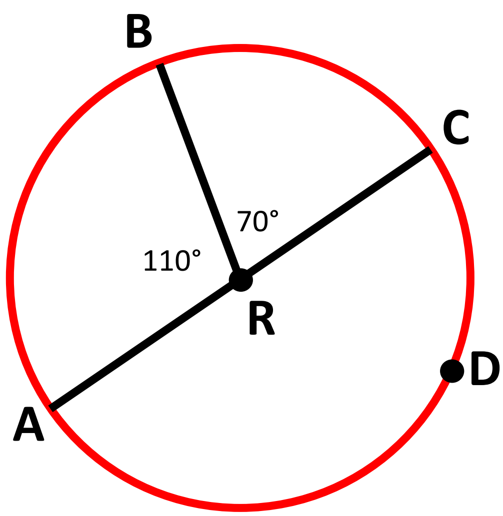
an angle whose vertex is the center of the circle



$\angle$ACB is a central angle of circle C.

Minor arc – corresponding central angle is less than 180°
Major arc – corresponding central angle is greater than 180°

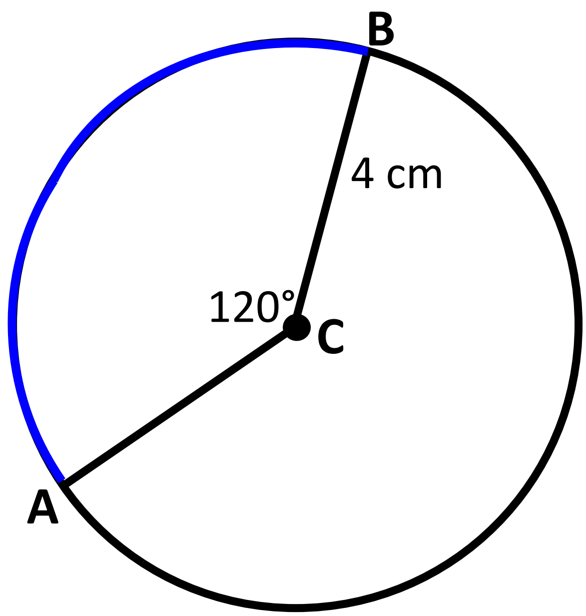
Measuring Arcs



Minor arcs	Major arcs	Semicircles
$m\overparen{AB} = 110°$	$m\overparen{BDA} = 250°$	$m\overparen{ADC} = 180°$
$m\overparen{BC} = 70°$	$m\overparen{BAC} = 290°$	$m\overparen{ABC} = 180°$

The measure of the entire circle is 360°.
The measure of a minor arc is equal to its central angle.
The measure of a major arc is the difference between 360° and the measure of the related minor arc.

Arc Length



$$\frac{\text{arc length}}{2\pi r} = \frac{\text{central angle}}{360°}$$

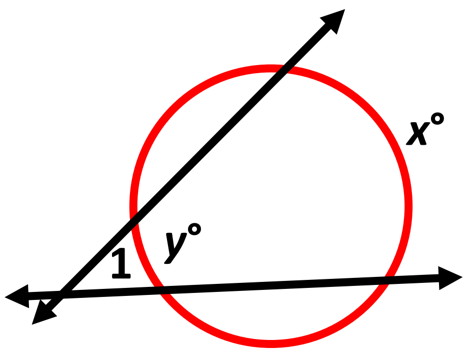
Example:

$$\frac{\text{arc length of } \overset{\frown}{AB}}{2\pi \cdot 4} = \frac{120°}{360°}$$

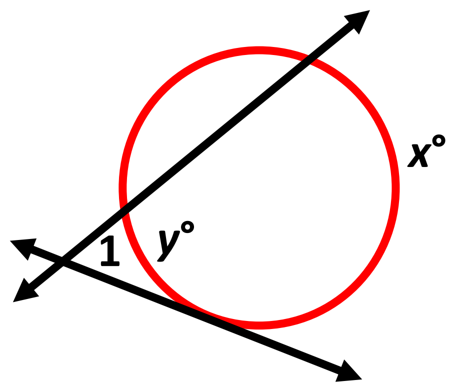
$$\text{arc length of } \overset{\frown}{AB} = \frac{8}{3}\pi \text{ cm}$$

Secants and Tangents

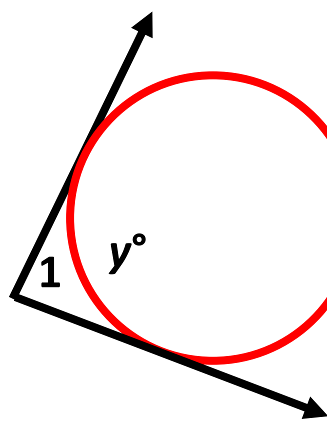
Two secants



Secant-tangent



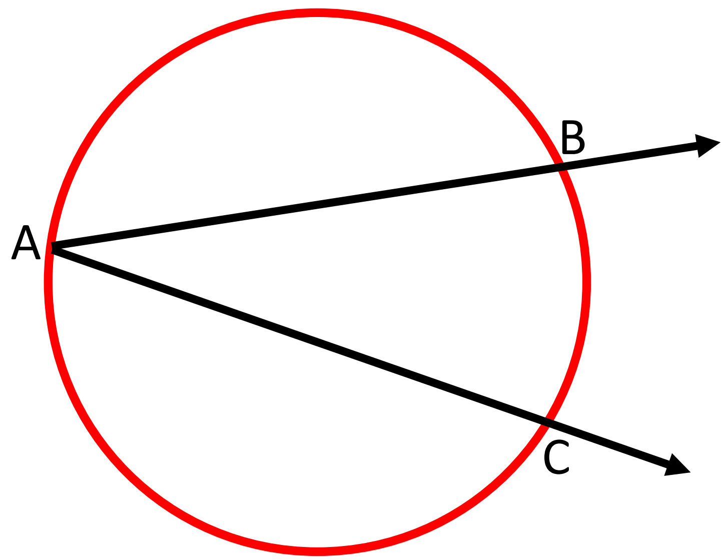
Two tangents



$$m\angle 1 = \frac{1}{2}(x^\circ - y^\circ)$$

Inscribed Angle

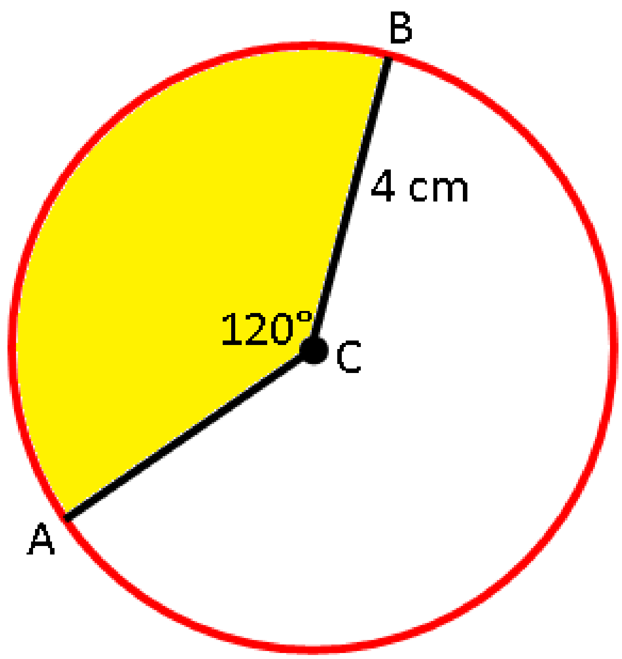
angle whose vertex is a point on the circle and whose sides contain chords of the circle



$$m\angle BAC = \frac{1}{2} m\overset{\frown}{BC}$$

Area of a Sector

region bounded by two radii and their intercepted arc



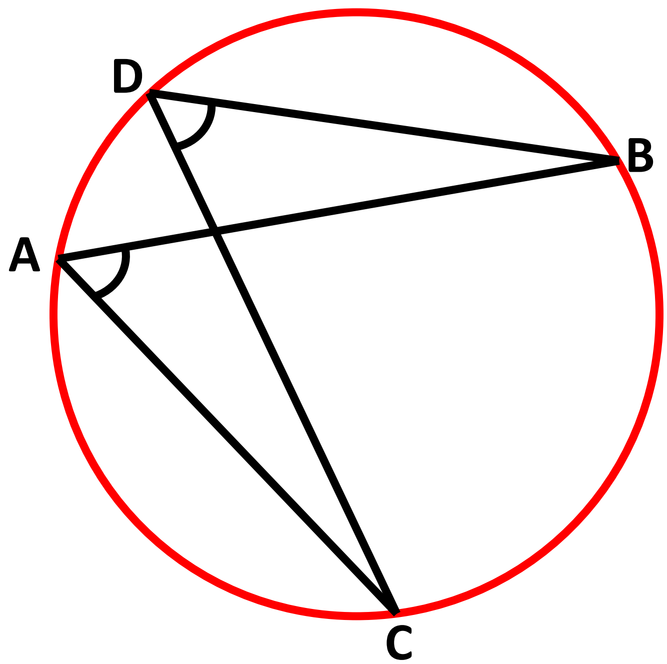
$$\frac{\text{area of sector}}{\pi r^2} = \frac{\text{measure of intercepted arc}}{360°}$$

Example:

$$\frac{\text{area of sector ACB}}{\pi \cdot 4^2} = \frac{120°}{360°}$$

$$\text{area of sector ACB} = \frac{16}{3}\pi \text{ cm}$$

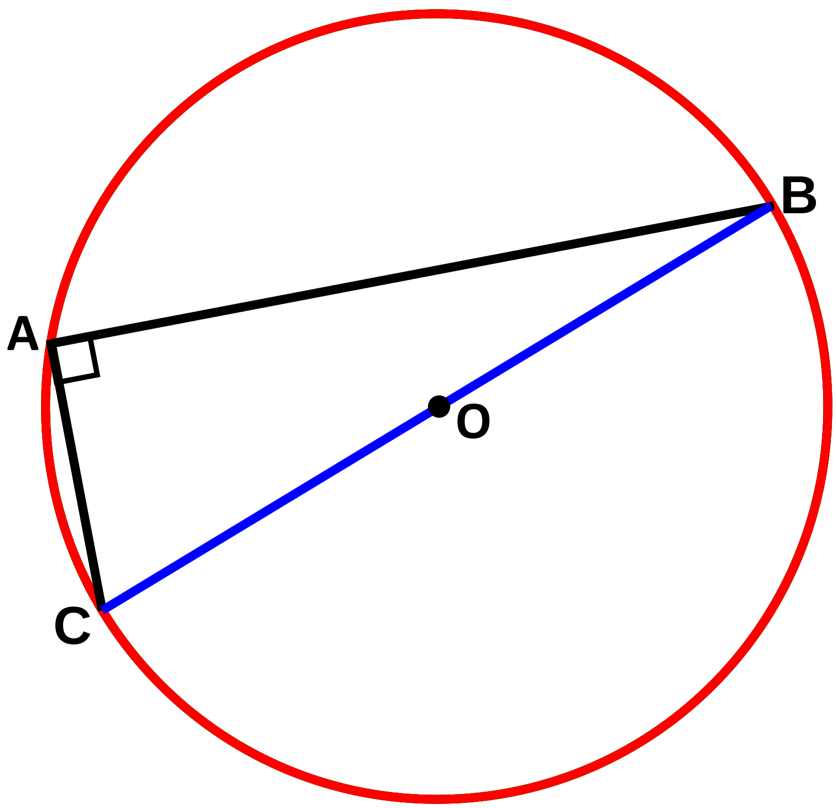
Inscribed Angle Theorem



If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

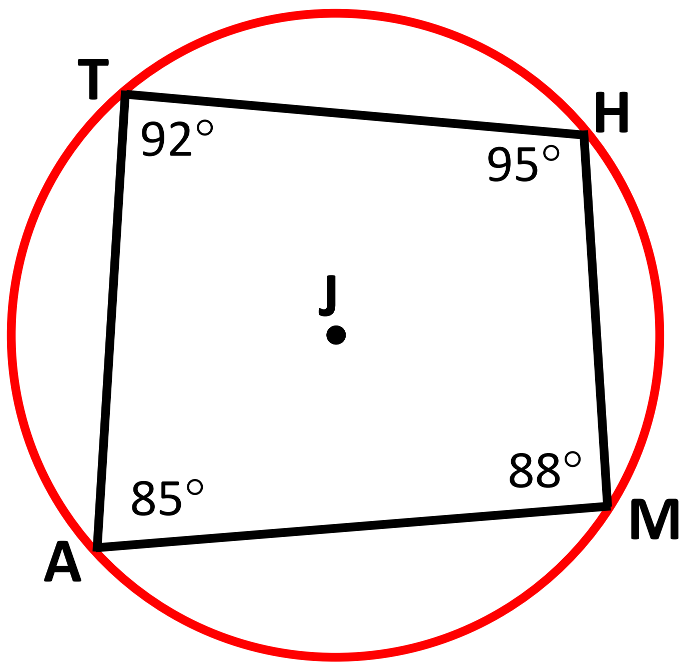
$\angle BDC \cong \angle BAC$

Inscribed Angle Theorem



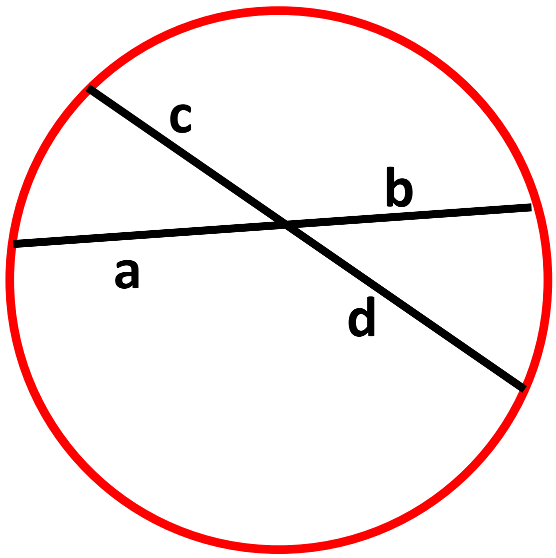
m∠BAC = 90° if and only if $\overline{BC}$ is a diameter of the circle.

Inscribed Angle Theorem



M, A, T, and H lie on circle J if and only if
$m\angle A + m\angle H = 180°$ and
$m\angle T + m\angle M = 180°$.

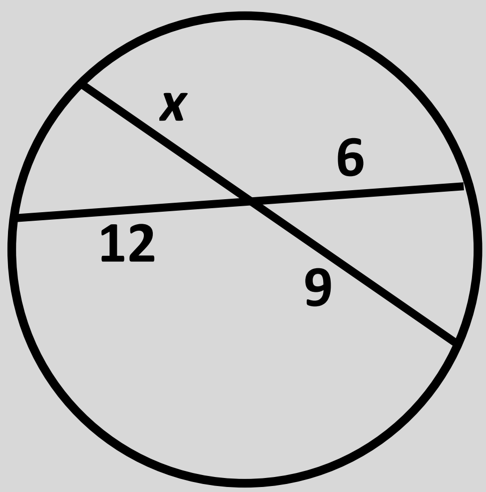
Segments in a Circle



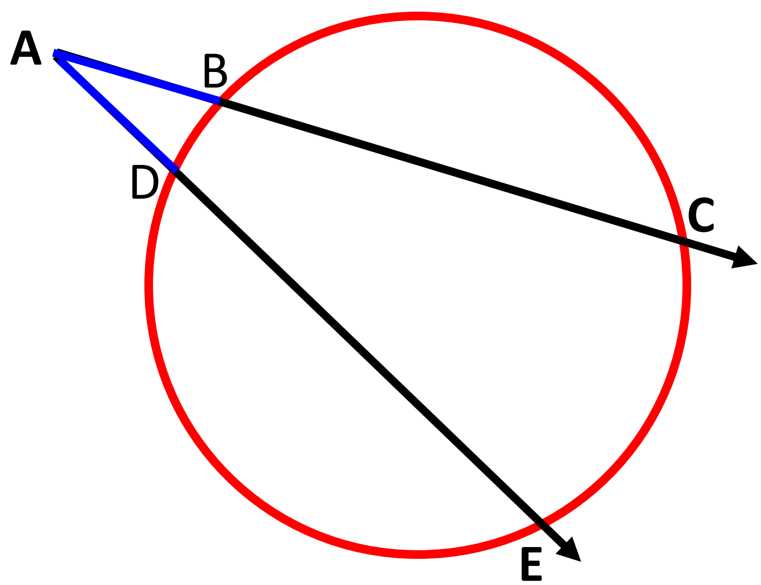
If two chords intersect in a circle, then a·b = c·d.

Example:

$$12(6) = 9x$$
$$72 = 9x$$
$$8 = x$$

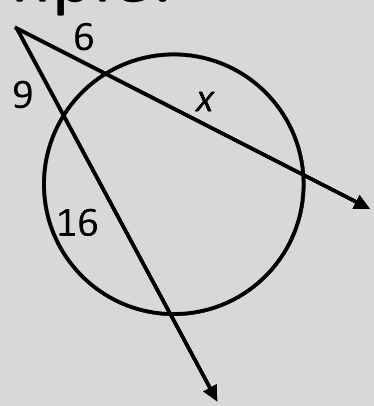


Segments of Secants Theorem



$AB \cdot AC = AD \cdot AE$

Example:

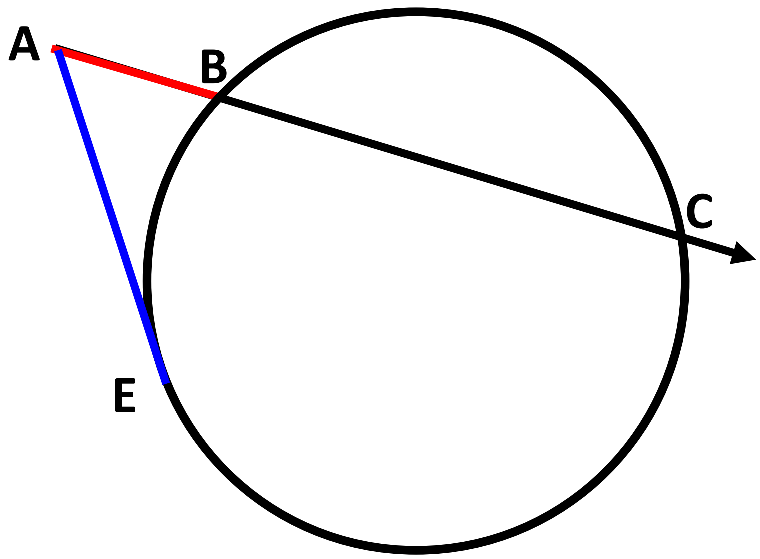


$6(6 + x) = 9(9 + 16)$

$36 + 6x = 225$

$x = 31.5$

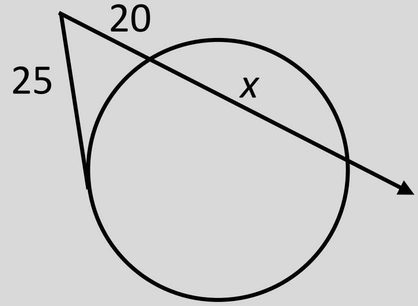
Segments of Secants and Tangents Theorem



$$AE^2 = AB \cdot AC$$

Example:

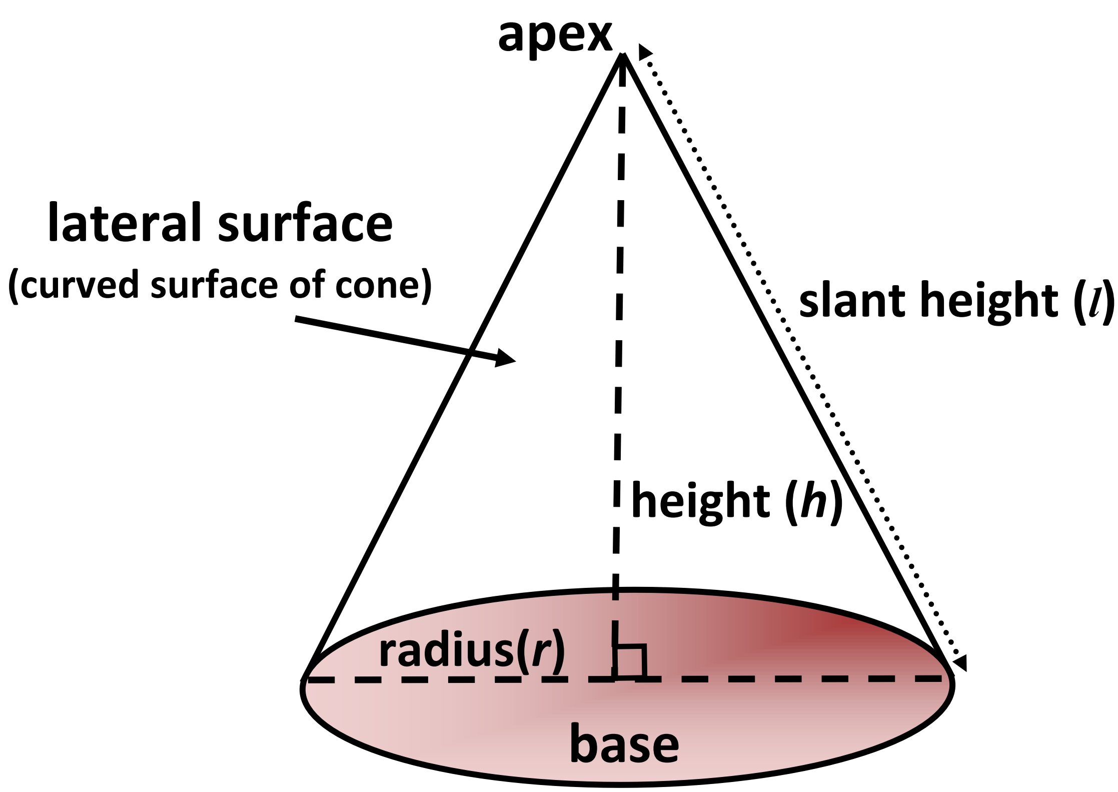
$$25^2 = 20(20 + x)$$
$$625 = 400 + 20x$$
$$x = 11.25$$



Three-Dimensional Figures

Cone

solid that has a circular base, an apex, and a lateral surface



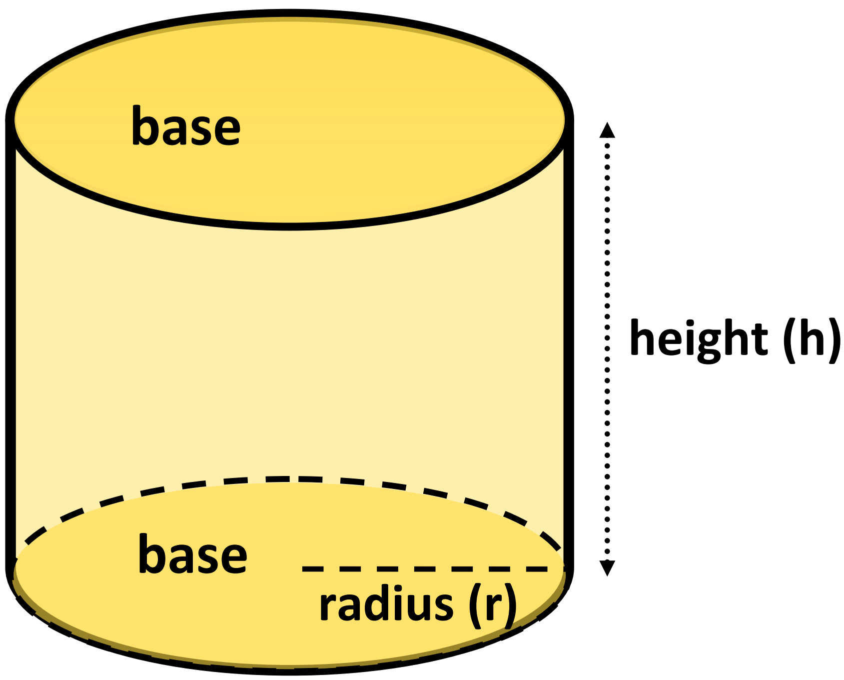
$$V = \frac{1}{3}\pi r^2 h$$

L.A. (lateral surface area) $= \pi r l$

S.A. (surface area) $= \pi r^2 + \pi r l$

Cylinder

solid figure with congruent circular bases that lie in parallel planes



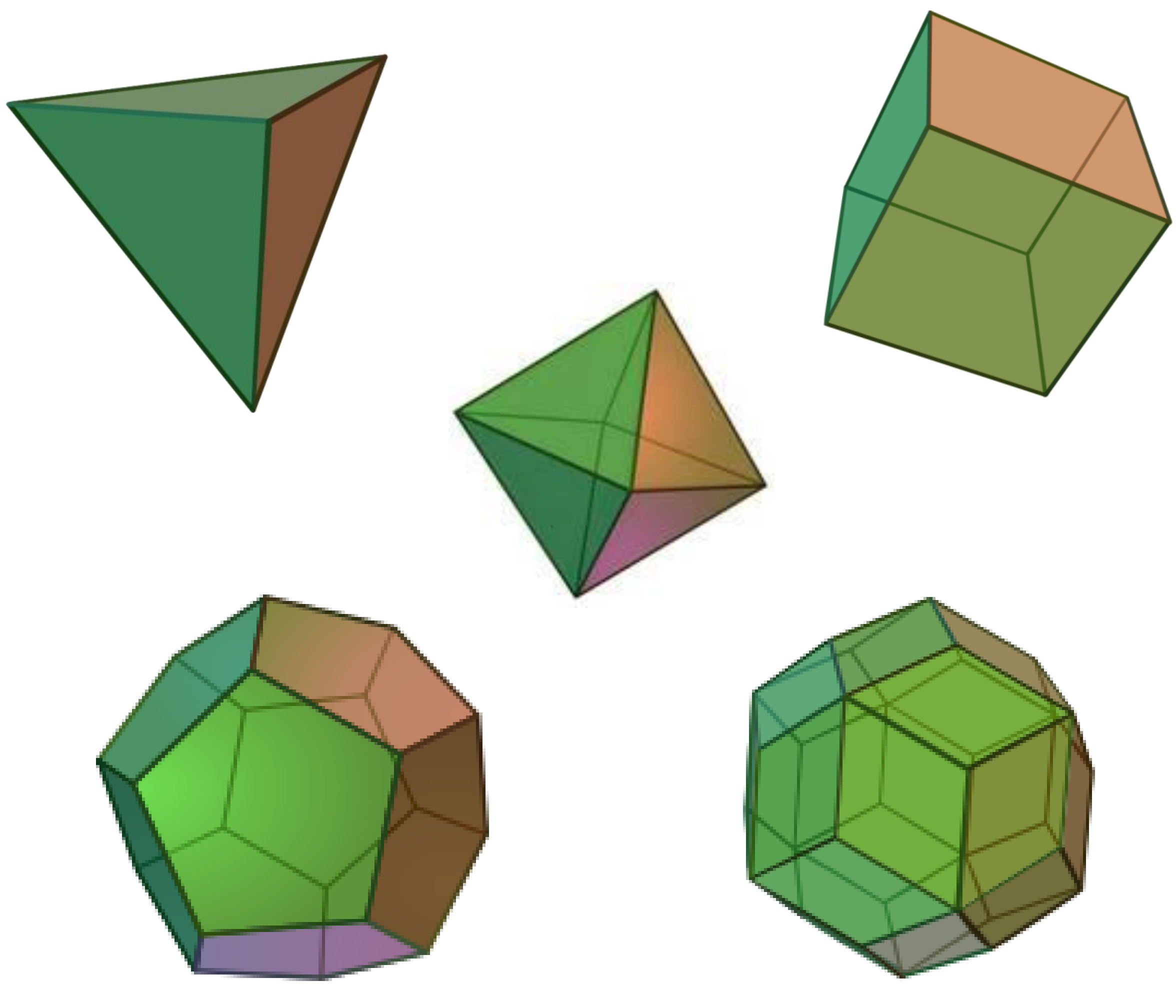
$$V = \pi r^2 h$$

L.A. (lateral surface area) $= 2\pi rh$

S.A. (surface area) $= 2\pi r^2 + 2\pi rh$

Polyhedron

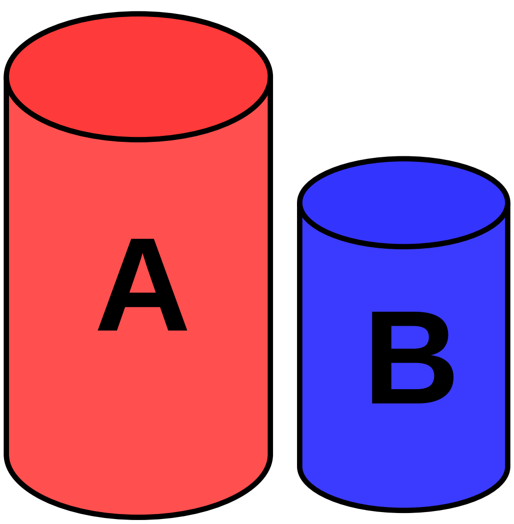
solid that is bounded by polygons, called faces



Similar Solids Theorem

If two similar solids have a scale factor of **a**:**b**, then their corresponding surface areas have a ratio of $\mathbf{a^2}$: $\mathbf{b^2}$, and their corresponding volumes have a ratio of $\mathbf{a^3}$: $\mathbf{b^3}$.

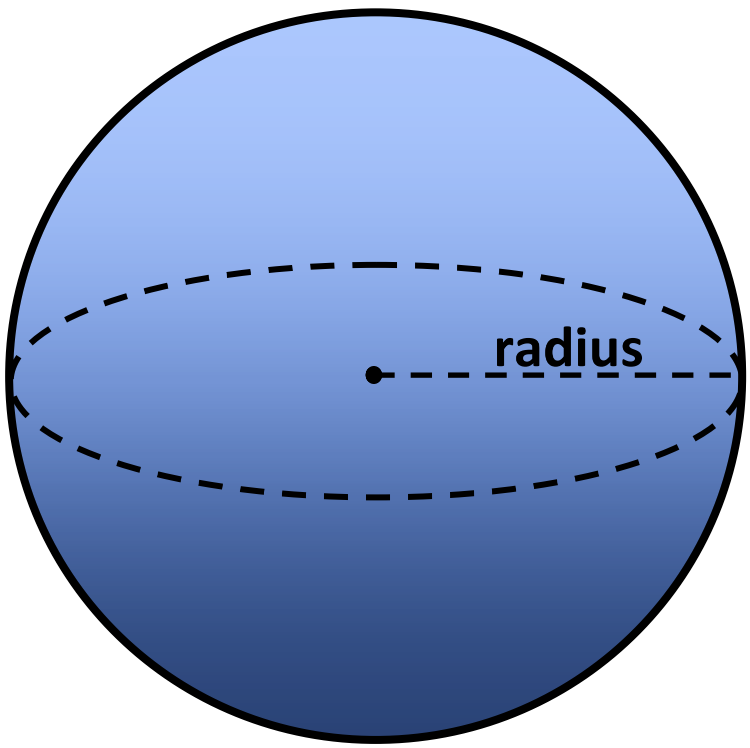
cylinder A ~ cylinder B



Example		
scale factor	**a : b**	**3:2**
ratio of surface areas	$\mathbf{a^2}$: $\mathbf{b^2}$	**9:4**
ratio of volumes	$\mathbf{a^3}$: $\mathbf{b^3}$	**27:8**

Sphere

a three-dimensional surface of which all points are equidistant from a fixed point

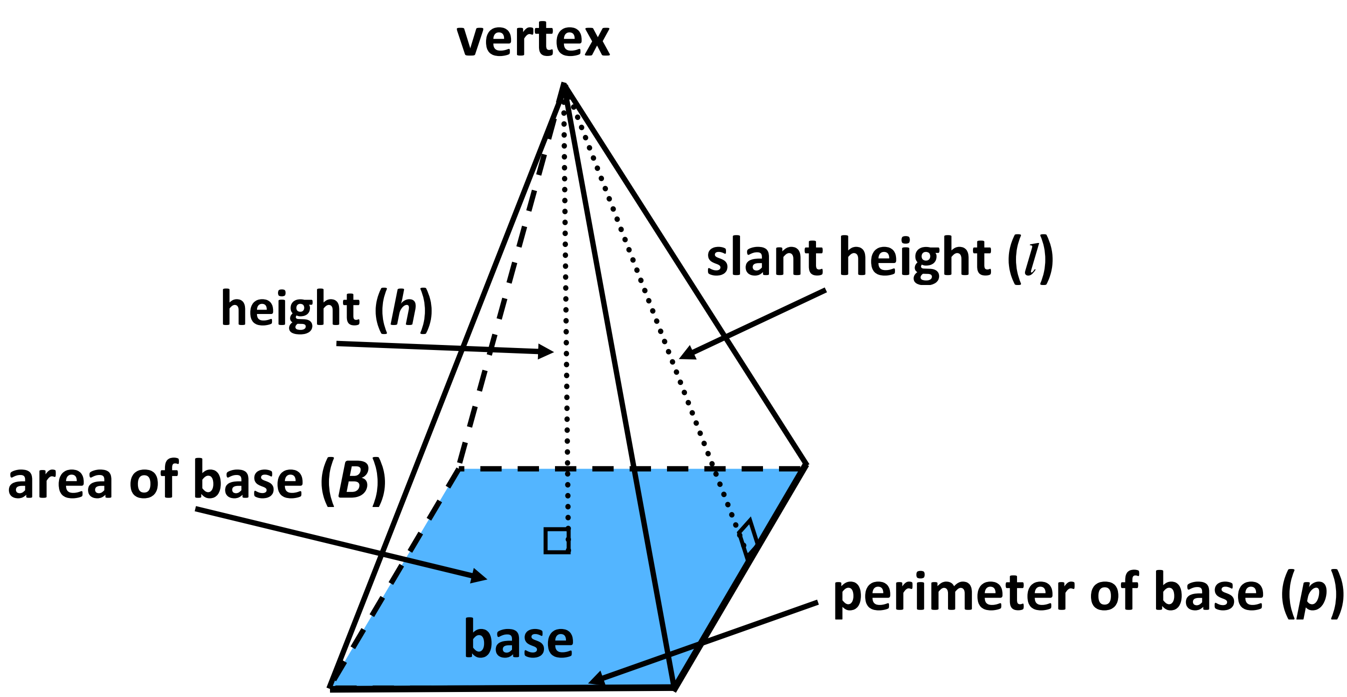


$$V = \frac{4}{3}\pi r^3$$

S.A. (surface area) $= 4\pi r^2$

Pyramid

polyhedron with a polygonal base and triangular faces meeting in a common vertex



$$V = \frac{1}{3}Bh$$

$$\text{L.A. (lateral surface area)} = \frac{1}{2}lp$$

$$\text{S.A. (surface area)} = \frac{1}{2}lp + B$$